

Mirage resolution of cosmological singularities

Frédéric Leblond

*Enrico Fermi Institute, University of Chicago,
5640 S. Ellis Av., Chicago, IL 60637, USA*
fleblond@theory.uchicago.edu

ABSTRACT: We study time-dependent backgrounds in the low energy regimes of string theories. In particular the emphasis is on the general study of exotic phenomena such as positive acceleration and gravitational bounces. We generalize the usual Hawking–Penrose cosmological singularity theorems to higher-dimensional spacetimes and discuss their implications for time-dependent solutions in supergravity theories. The explicit examples we consider fall in two categories. First we consider effective lower-dimensional gravitational theories obtained from compactifications of ten and eleven-dimensional supergravity. We argue and explain why non-singular solutions (*e.g.*, with positive acceleration and possibly a bounce) can in principle be obtained. However we show that their uplift to higher dimensions is always singular as predicted by the theorems. Secondly we revisit the issue of supergravity s-branes. Our main result is to propose a generic mechanism by which the usual singularities can be resolved.

KEYWORDS: cosmology, s-branes, singularities, supergravity.

Contents

1. Introduction	2
2. Background notions	3
2.1 Equations of motion	3
2.2 Attraction and energy conditions	5
2.3 Cosmological singularity theorems	8
3. Geometrical bounces and singularity resolution	9
3.1 Local dynamics of a bounce	9
3.2 Solitary bounces and energy conditions	10
3.3 The compulsory singularity	11
3.4 Singular Hubble loops	12
4. Spacetimes with positive acceleration	15
4.1 Bounces and FLRW cosmology	15
4.2 The infinite throat	18
4.3 Bouncing spacetimes	22
5. Supergravity applications	24
5.1 Flux compactifications	24
5.2 Effective potential and scalar field dynamics	26
5.3 The arduous ascension	26
6. Spacelike branes	27
6.1 The s8-brane and the rolling tachyon	28
6.2 Unstable branes within branes	29
6.2.1 Asymptotically dS non-singular s-branes	30
6.2.2 The other s-branes	32
6.3 Open-closed duality and quantum effects	33
7. Discussion	33
A. Scalar field inter-breeding	35

1. Introduction

Perhaps the most pressing problem in theoretical physics is to explain the current state of acceleration in the universe [1]. Related to this is the fact that there is no candidate theory of quantum gravity providing a consistent mechanism associated with the generation of a positive cosmological constant (see, however, ref. [2]). Another outstanding problem consists in our inability to convincingly resolve cosmological singularities.¹ The purpose of this paper is to explore further the nature of cosmological singularities and positive acceleration in the low energy limits of string theories. This means our work is relevant in the context of the ten-dimensional and eleven-dimensional supergravity theories. For ten-dimensional supergravity as long as the scales involved are considerably larger than the string length (and that the string coupling is kept small), the corresponding low-energy actions will capture relevant time-dependent physics. More specifically we are considering ten- or eleven-dimensional Einstein gravity coupled to sources found in supergravity theories. The latter will consist in massless fields such as the dilaton and the Ramond–Ramond forms as well as extended sources such as p -branes.

A related motivation for our work was the study of gravitational throats. These are spacetime regions associated with a direction along which a spatial volume element goes through a minimum.² This is illustrated on figure 1 for a spherical hypersurface. If the minimum is reached via a timelike direction the resulting geometry is a bouncing cosmology. When the extremum is the result of a contraction followed by an expansion along a spacelike direction this is a wormhole.

Early work on Euclidean wormholes focused on their potential role in a theory of quantum gravity based on the Euclidean path integral formalism (see, *e.g.*, refs. [5, 6, 7]). A recent attempt [8] (see also ref. [9]) was made to consider the role played by these configurations in gauge/gravity dualities. Four-dimensional Lorentzian wormhole solutions were found in the past (see, *e.g.*, refs. [10, 11]). It was then shown that the existence of such a gravitational throat requires that the matter supporting it violates the weak energy condition. In section 2.2 we derive and explain the gravitational energy conditions. There are also topological censorship theorems [12] showing that wormholes cannot exist unless their disconnected boundaries are separated by an horizon.

In this paper we primarily study bouncing cosmologies. The generalization of our results to Lorentzian wormholes is presented in the discussion section. It is a well-known fact that time-dependent gravitational bounces can only be supported by matter sources violating the strong energy condition. In section 2.3 we generalize to higher dimensions the Hawking–Penrose singularity theorems. This ensures that theories with sources respecting the latter condition do not admit non-singular time-dependent solutions. However we point out that this conclusion does not necessarily hold in higher-dimensional spacetimes where only a sub-manifold is bouncing. An example of this would be if the geometry on this sub-space is that of de Sitter space in global coordinates. In section 3 we consider this phenomenon in details. The gravitational theory on the bouncing sub-manifold will appear regular but this is only an illusion since the theorems predict that singular points must develop as seen from the higher-dimensional geometry point of view. Another possibility is that rather than being associated with a bounce the sub-manifold is a non-singular forever expanding (or forever contracting) spacetime. This is a feature associated with the representation of de Sitter space in inflationary coordinates. The region

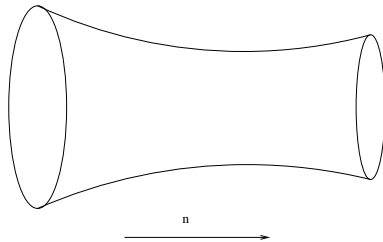


Figure 1: Schematic depiction of a spherical gravitational throat. If n is a timelike vector the corresponding geometry is a bouncing cosmology. Cases for which n is spacelike are associated with Lorentzian or Euclidean wormholes.

¹However interesting attempts were made in the context of string theory (see refs. [3, 4]).

²This simple definition lifts the restriction that a throat must be associated with a compact sub-manifold. For instance when considering time-dependent homogeneous and isotropic FLRW cosmology the term gravitational throat is not only relevant for a spherical foliation.

where the volume of the spacetime vanishes is then a non-singular horizon.

In section 4 we consider homogeneous and isotropic cosmological solutions of $(p+1)$ -dimensional Einstein gravity ($p \leq 10$) coupled to a scalar field with positive exponential potential. We find analytic solutions for spacetimes with flat foliations. We obtain their asymptotic behavior and show these geometries are always associated with an intermediary phase of positive acceleration. Although the scalar field is allowed to violate the strong energy condition we find there is always a curvature singularity either in the past or the future when the scale factor becomes small. Then we consider the corresponding spacetimes with positive spatial curvature. We find interesting non-singular bouncing cosmologies but show that if the slope parameter for the potential is too large the geometries are singular. Then in section 5 we consider flux compactifications of ten- and eleven-dimensional supergravity on maximally symmetric spaces. This provides a natural way to embed the $k = 0$ and $k = +1$ lower-dimensional spacetimes studied in section 4. We discuss the resulting geometries in the context of singularity resolution from the point of view of a lower-dimensional observer.

Finally in section 6 we apply our results to the study of supergravity s-branes. We use the singularity theorems to show that sp -brane with $p \leq 7$ are always singular. The $s8$ -brane evades the theorems but we show that it is nevertheless singular. When studying unstable branes from a gravitational perspective we have always assumed that the end point (and the time when brane formation starts) of the decay is the closed string vacuum with vanishing energy density. We show that considering a more general setup where unstable branes evolve inside larger stable branes might generically lead to a resolution of all previously found singularities.

2. Background notions

2.1 Equations of motion

We begin by deriving the general relativistic equations of motion that will be used throughout the paper. We choose to write these down in terms of the extrinsic and intrinsic curvatures. Not only does this simplify the analysis but it allows us to address general issues (*e.g.*, the role of inhomogeneity and anisotropy) which are hard to take into account when a more specific metric ansatz is used. Beginning with section 3.4 our analysis will be in terms of a more intuitively accessible homogeneous metric ansatz.

The most general form for the metric of a $(n+1)$ -dimensional gravitational background is

$$ds^2 = -(N^2 - N_i N^i) dt^2 + 2N_j dx^j dt + {}^{(n)}g_{ij} dx^i dx^j, \quad (2.1)$$

where $i, j = 1, \dots, n$ and $N = N(t, x^i)$, $N^i = N^i(t, x^i)$ are respectively the lapse function and the shift functions. The spacetime is assumed to be globally hyperbolic³ with the geometry of the constant t hypersurfaces characterized by the spatial metric ${}^{(n)}g_{ij}$. All information about the intrinsic curvature of the Cauchy surfaces is contained in the Riemann tensor components ${}^{(n)}R^i{}_{jkl}$. The extrinsic curvature for constant t is given by the expression

$$K_{ik} = \frac{1}{2N} \left[N_{i|k} + N_{k|i} - \frac{\partial g_{jk}}{\partial t} \right], \quad (2.2)$$

and the volume element for the spacetime is $\sqrt{-{}^{(n+1)}g} dx^n dt = N \sqrt{{}^{(n)}g} dx^n dt$. The notation $|k$ represents the covariant derivative along the spatial direction labelled k .

An interesting class of time-dependent solutions would consist in spacetimes that are interpolating between two different vacua. The boundary (at conformal infinity) of these spacetimes would

³This assumption will be momentarily relaxed when we consider singularity theorems in section 2.3.

then be disconnected. This is the time-dependent equivalent of static (Lorentzian or Euclidean) wormholes. Provided the matter sources generating these time-dependent geometries satisfy certain (reasonable) energy conditions, it is clear that such spacetimes cannot exist (this is detailed in section 3). We will see that vacua interpolating spacetimes can be realized only if something prevents the formation of singular points. From the point of view of the gravitational field such a singularity resolution mechanism could, for example, take the form of a bounce or something that stabilizes the scale of the spatial sections to finite size. Another possibility is that the volume of the Cauchy surfaces could be eternally contracting (or expanding) in such a way that the spacetime does not develop singular points and therefore remains geodesically complete (non-singular big-bang or big-crunch). We find explicit examples of these phenomena in section 6 while in section 3 we consider in detail the dynamics of bounces. As will become clear shortly a minimal requirement for these mechanisms of singularity resolution to take effect is that positive acceleration be allowed by the matter sources in the system.

Finding analytic solutions of the general form (2.1) is very difficult. However to investigate the possibility of getting bouncing geometries and spacetimes with positive acceleration the first step consists in studying the local dynamics (global aspects are considered in section 2.3). We therefore use Gaussian normal coordinates to write down the metric locally in the form

$$ds^2 = -dn^2 + {}^{(n)}g_{ij}dx^i dx^j, \quad (2.3)$$

where n is a timelike coordinate normal to the hypersurfaces with spatial metric ${}^{(n)}g_{ij}$. An example we consider in further detail later is that of an anisotropic foliation in the form of a product geometry. This assumes that the region close to the bounce is of the form

$$ds^2 = -dn^2 + {}^{(p)}g_{\hat{i}\hat{j}}dx^{\hat{i}}dx^{\hat{j}} + {}^{(n-p)}g_{ab}dx^a dx^b, \quad (2.4)$$

where $\hat{i}, \hat{j} = 1, \dots, (n-p)$ and $a, b = (n-p+1), \dots, n$. This is relevant to the study of gravitational fields generated by unstable D-branes [13, 14, 15, 16, 17, 18] which is expected if we follow our intuition gained studying regular static branes [19]. In this case the spacetime close to the core of the object is of the form eq. (2.4) with n a spatial coordinate and ${}^{(n)}g_{\hat{i}\hat{j}}$ a Lorentzian metric.

We now proceed and write down the Einstein equations associated with the metric (2.3). We begin by using the Gauss-Codazzi equation (see, *e.g.*, ref. [21]) to express the relevant Riemann tensor components in terms of n -dimensional quantities, *i.e.*, the intrinsic and the extrinsic curvatures of the spatial hypersurfaces,

$${}^{(n+1)}R^m_{ijk} = {}^{(n)}R^m_{ijk} + \frac{1}{n^2} [K_{ij}K^m_k - K_{ik}K^m_j], \quad (2.5)$$

$${}^{(n+1)}R^n_{ijk} = -\frac{1}{n^2} [K_{ij|k} - K_{ik|j}], \quad (2.6)$$

$${}^{(n+1)}R^n_{ink} = \frac{1}{n^2} \left[\frac{\partial K_{ik}}{\partial n} + K_{im}K^m_k \right], \quad (2.7)$$

where n is the timelike vector normal to the hypersurfaces.⁴ It is then straightforward to write down the Ricci tensor components using eqs. (2.5)–(2.7),

$${}^{(n+1)}R_{ik} = {}^{(n)}R_{ik} + \frac{1}{n^2} \left[\frac{\partial K_{ik}}{\partial n} + 2(K^2)_{ik} - K_{ik}\text{Tr} K \right], \quad (2.8)$$

$${}^{(n+1)}R_{ni} = \frac{g_{nn}}{n^2} \left[-K^{[k}_{ik} + (\text{Tr} K)_{|i} \right], \quad (2.9)$$

⁴The spacetimes of interest are time orientable which implies that we can define a smooth non-vanishing timelike vector everywhere.

$${}^{(n+1)}R_{nn} = \frac{g_{nn}}{n^2} \left[\frac{\partial \text{Tr } K}{\partial n} - \text{Tr } K^2 \right], \quad (2.10)$$

where $g_{nn} = -1$ and $n^2 = -1$. Finally the Ricci scalar is found to be

$${}^{(n+1)}R = {}^{(n)}R + \frac{1}{n^2} \left[2 \frac{\partial \text{Tr } K}{\partial n} - \text{Tr } K^2 - (\text{Tr } K)^2 \right]. \quad (2.11)$$

We are now in a position to write down the Einstein equations,

$${}^{(n+1)}G_{\mu\nu} = {}^{(n+1)}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} {}^{(n+1)}R = 8\pi G_N T_{\mu\nu} \quad (2.12)$$

$(\mu, \nu = 0, 1, \dots, n)$, in terms of the intrinsic and extrinsic curvature of the Cauchy surfaces,

$$8\pi G_N T_{ik} = {}^{(n+1)}G_{ik} = {}^{(n)}G_{ik} + \frac{1}{n^2} \left[\frac{\partial K_{ik}}{\partial n} - K_{ik} \text{Tr } K + 2 (K^2)_{ik} - g_{ik} \frac{\partial \text{Tr } K}{\partial n} + \frac{1}{2} g_{ik} \text{Tr } K^2 + \frac{1}{2} g_{ik} (\text{Tr } K)^2 \right], \quad (2.13)$$

$$8\pi G_N T_{ni} = {}^{(n+1)}G_{ni} = \frac{g_{nn}}{n^2} \left[-K_{ik}^{[k} + (\text{Tr } K)_{|i} \right], \quad (2.14)$$

$$8\pi G_N T_{nn} = {}^{(n+1)}G_{nn} = -\frac{1}{2}g_{nn} {}^{(n)}R + \frac{g_{nn}}{n^2} \left[-\frac{1}{2} \text{Tr } K^2 + \frac{1}{2} (\text{Tr } K)^2 \right]. \quad (2.15)$$

The quantity G_N is the $(n+1)$ -dimensional Newton constant. It is important to recall that strictly speaking the equations of motion we derived are valid only locally when the metric is written in Gaussian normal coordinates. However beginning with section 3.4 we will consider, for simplicity, homogeneous (but anisotropic) global geometries of the form (2.3).

Let us be more precise with respect to the correct procedure for finding solutions associated with the equations derived above. We want to bring these down to a system of (at most) $2 \times n(n+1)/2$ first order differential equations in ${}^{(n)}g_{ik}$ and K_{ik} . Using eqs. (2.2) and (2.3) we can write

$$K_{ik} = -\frac{1}{2} \frac{\partial g_{ik}}{\partial n}. \quad (2.16)$$

From eq. (2.8) we have

$$\frac{\partial K_{ik}}{\partial n} = {}^{(n)}R_{ik} - [2 (K^2)_{ik} - K_{ik} \text{Tr } K] - 8\pi G_N \left[T_{ik} - \frac{g_{ik}}{n-1} T \right]. \quad (2.17)$$

Eqs. (2.16) and (2.17) act as evolution equations respectively for ${}^{(n)}g_{ik}$ and K_{ik} . This system of $n(n+1)$ first order differential equations must be supplemented by the evolution equations associated with the matter fields. It may also be verified that if eq. (2.15) is satisfied on some initial constant t hypersurface then it will hold at all times by virtue of the equations of motion. Therefore eq. (2.15) acts as a constraint on the initial values of ${}^{(n)}g_{ik}$, K_{ik} and the first derivatives of the matter fields acting as sources. For example in order to find vacuum solutions to the Einstein equations we must set $T_{\mu\nu}$ to zero and solve eqs. (2.16) and (2.17) while making sure the initial conditions satisfy the constraint eq. (2.15).

2.2 Attraction and energy conditions

So far we have considered the equations governing the spacetime curvature given arbitrary sources. In this section we review the effects that the resulting curvature will have on the behavior of geodesics of physical interest. This will be useful in section 2.3 where we review singularity theorems relevant to cosmology in $(n+1)$ dimensions.

We consider a smooth congruence of timelike geodesics parametrized with the affine parameter τ .⁵ The associated vector field ξ^μ is normalized such that $\xi^2 = -1$. We introduce the spatial vector field $B_{\mu\nu} = \nabla_\nu \xi_\mu$, the symmetric part of which is related to the extrinsic curvature through $K_{ij} = -B_{(ij)}$. Then we consider a smooth one-parameter sub-family $\gamma_s(\tau)$ of geodesics on the congruence and we let η^μ represent an infinitesimal spatial displacement from γ_0 to a nearby geodesic within the sub-family. It is easy to see that $\xi^\mu \nabla_\mu \eta^\nu = B_\mu^\nu \eta^\mu$, which implies that B is a linear map measuring how much an observer on γ_0 would see the nearby geodesics being stretched and rotated. Introducing the spatial metric $h_{\mu\nu} = g_{\mu\nu} + \xi_\mu \xi_\nu$, we decompose B into symmetric-traceless, anti-symmetric and scalar parts,

$$B_{\mu\nu} = \sigma_{\mu\nu} + \omega_{\mu\nu} + \frac{1}{n}\theta h_{\mu\nu}. \quad (2.18)$$

These different components are given by

$$\sigma_{\mu\nu} = B_{(\mu\nu)} - \frac{1}{n}\theta h_{\mu\nu}, \quad \omega_{\mu\nu} = B_{[\mu\nu]}, \quad \theta = B^{\mu\nu} h_{\mu\nu} = B, \quad (2.19)$$

which respectively correspond to the shear, the twist and the expansion of the congruence. The equations governing the time evolution of this tensor are shown to be

$$\xi^\mu \nabla_\nu B_{\lambda\rho} = -B_\rho^\mu B_{\lambda\mu} + R_{\mu\rho\lambda}^\kappa \xi^\mu \xi_\kappa. \quad (2.20)$$

For us the most relevant component is the trace of eq. (2.20),

$$\xi^\mu \nabla_\mu \theta = \frac{d\theta}{d\tau} = -\frac{1}{n}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}\xi^\mu \xi^\nu. \quad (2.21)$$

This is the Raychaudhuri equation which describes the rate of expansion of nearby geodesics in a congruence.⁶ From now on we set $\omega_{\mu\nu} = 0$, *i.e.*, we consider hypersurface orthogonal spacetimes only. The analysis presented for timelike geodesics can be repeated in the case of null geodesics by introducing the tensor $\hat{B}_{\mu\nu} = \nabla_\nu k_\mu$, where k^μ is a null vector. It is straightforward to derive the equation governing the expansion of null geodesics $\gamma_s(\lambda)$ in a congruence,

$$\frac{d\theta}{d\lambda} = -\frac{1}{n-1}\theta^2 - \hat{\sigma}_{\mu\nu}\hat{\sigma}^{\mu\nu} + \hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu. \quad (2.22)$$

Our point of view (essentially following that presented in ref. [22]) is that energy conditions on matter sources impose that gravity is attractive which is equivalent to requiring that

$$\frac{d\theta}{d\tau} \geq 0 \quad \text{and} \quad \frac{d\theta}{d\lambda} \geq 0 \quad (2.23)$$

everywhere. In the case of null geodesics this corresponds to requiring that all sources satisfy

$$T_{\mu\nu}k^\mu k^\nu \geq 0. \quad (2.24)$$

This is called the null energy condition (NEC) and is enough to insure, for example, that converging null rays will never re-expand.

It is believed that any physically reasonable system is associated with a stress-energy tensor that can be diagonalized.⁷ It will therefore be useful to consider systems of the form

$$T_{\mu\nu} = (\rho, g_{i_1 j_1} p_1, \dots, g_{i_N j_N} p_N), \quad (2.25)$$

⁵Our approach in this subsection is simply to generalize results from chapter 8 of ref. [22] to $(n+1)$ dimensions.

⁶The symmetric trace-free part of eq. (2.20) governs the dynamics of $\sigma_{\mu\nu}$ while the anti-symmetric components would give information about $\omega_{\mu\nu}$.

⁷A notable exception being the example of a null fluid (see ref. [22]).

where ρ is the energy density and the p_i 's are normal pressures ($i = 1, \dots, N$). Then the NEC corresponds to

$$\rho + p_i \geq 0 \quad \forall i, \quad (2.26)$$

with no constraint on the sign of ρ . The quantity $T_{\mu\nu}\xi^\mu\xi^\nu$ physically represents the energy density of matter as measured by an observer whose $(n+1)$ -velocity is ξ^μ . The weak energy condition (WEC) corresponds to the requirement that $T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ which supersedes the NEC by constraining the energy density to be positive ($\rho \geq 0$). Now the strong energy condition (SEC) corresponds to the statement that

$$R_{\mu\nu}\xi^\mu\xi^\nu = 8\pi G_N \left[T_{\mu\nu}\xi^\mu\xi^\nu - \frac{1}{n-1}(\text{Tr } T)\xi^\mu\xi_\mu \right] \geq 0, \quad (2.27)$$

which implies that eq. (2.26) is satisfied, $\rho \geq 0$ and

$$\left(\sum_{i=1}^N n_i - 2 \right) \rho + \sum_{i=1}^N n_i p_i \geq 0, \quad (2.28)$$

where n_i is the number of spatial directions with normal pressure p_i . For a perfect fluid in four dimensions the latter inequality becomes the familiar $\rho + 3p \geq 0$. This condition plays a crucial role in the derivation of Hawking–Penrose singularity theorems [23] relevant for cosmology (see section 2.3 for a review). Clearly de Sitter space does not respect this energy condition as manifested by the bounce present in its global representation (see, *e.g.*, ref. [24]).

The energy conditions presented so far are not fundamental since they have not been derived from first principle in any theory containing gravity. However it is interesting (and important for the problems of interest here) to note that any excitation of the massless bosonic closed string fields in the ten- or eleven-dimensional supergravity theories respect the SEC. A notable exception is the 9-form field in massive Type IIA supergravity (see section 6 for an application). Also there are non-perturbative objects in string theory which violate the SEC. In particular, p -branes with $p \geq 8$ are repulsive with a stress-tensor of the form

$$T_{\mu\nu} = T_p(1, -1, \dots, -1)\delta(\mathbf{y}), \quad (2.29)$$

where \mathbf{y} represents the spatial directions transverse to the brane. In fact in the Newtonian limit the gravitational field sourced by these objects corresponds to

$$\nabla^2 \phi = 4\pi G_N [(n-2)\rho + pT_i^i], \quad (2.30)$$

where no sum is implied and ϕ is the usual classical gravitational potential. Using eq. (2.29) we find that the right-hand-side (RHS) of eq. (2.30) is negative (repulsive gravity) for $p > (n-2)$. In ten-dimensional supergravity this means that 8-branes (domain walls) and the space-filling 9-branes are repulsive. This is a general statement that applies to any co-dimension one or zero tensile object in a gravitational theory with any number of dimensions. Orientifold planes (negative tension objects) also violate the SEC for $p \leq 7$. The repulsive objects described here are static so we should naively not expect them to act as sources for time-dependent geometries (however see section 6).

The natural generalization of D-branes to time-dependent phenomena is to consider the unstable D-branes present in the non-perturbative spectrum [25]. In this case the instability is caused by the presence of an open string worldvolume tachyon in the perturbative spectrum. As shown in ref. [26] the stress-energy tensor associated with an homogeneous open string tachyon on an unstable Dp-brane is⁸

$$\rho = \frac{T_p}{2} (\cos(2\lambda\pi + 1)), \quad (2.31)$$

⁸These expressions are derived from a worldsheet approach on which the tachyon takes the form of the marginal boundary deformation: $\tilde{T}(t) = \lambda \cosh(t/\sqrt{2})$.

$$p = -T_p \left[\frac{1}{1 + e^{\sqrt{2}t} \sin^2(\lambda\pi)} + \frac{1}{1 + e^{-\sqrt{2}t} \sin^2(\lambda\pi)} - 1 \right], \quad (2.32)$$

where T_p is the brane tension. As expected if we consider the latter as a gravitational source it violates the SEC in ten dimensions for $p \geq 7$ just like the static Dp -branes.

Lastly we consider the dominant energy condition (DEC) which requires that $(-T^\mu_\nu \xi^\nu)$ is a future-directed timelike or null vector. For an observer with $(n+1)$ -velocity ξ^μ this vector measures the energy-momentum current of matter she observes. The DEC can be interpreted as a constraint imposing that the speed of energy flow of matter is less than the speed of light. It clearly makes sense physically for matter to possess this characteristic. If the sources are in the form of a perfect fluid, then the DEC implies that

$$\rho \geq |p_i| \quad \forall i. \quad (2.33)$$

We also note that the WEC is implied by this last condition.

2.3 Cosmological singularity theorems

Before we discuss the physics of bounces and cosmological acceleration, we briefly review some formal notions about singularities. If the SEC is satisfied then eq. (2.20) implies the inequality

$$\frac{d\theta^{-1}}{d\tau} \geq \frac{1}{n}, \quad (2.34)$$

which can be integrated to give

$$\theta^{-1}(\tau) \geq \theta_0^{-1} + \frac{\tau}{n}, \quad (2.35)$$

where θ_0 is the expansion rate on an arbitrary spatial hypersurface. If θ is negative (rays are converging) then eq. (2.20) implies that θ becomes infinite within $\Delta\tau = (n-1)/\theta_0$. This is a pathological behavior but to conclude that a spacetime is singular it is not enough to show that it contains conjugate points. For a spacetime to be called singular it must also contain maximal length curves.

In the rest of this paper we will be referring to two important singularity theorems.⁹ If the conditions stipulated in these theorems are satisfied then the corresponding spacetime is necessarily singular in the sense of timelike or null geodesic incompleteness. An important point is that the singularity theorems have nothing to say about the nature of the singularity. However they would appear to predict a breakdown of general relativity. For the first singularity theorem we paraphrase theorem 9.5.1 from ref. [22] where the proof can be found:

Theorem I: Suppose $(\mathcal{M}, g_{\mu\nu})$ is a globally hyperbolic spacetime with $R_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ for ξ^μ a timelike vector. If there exists a smooth spacelike Cauchy surface for which the trace of the extrinsic curvature is strictly negative ($-\theta = \text{Tr } K < 0$) everywhere, all past directed timelike geodesics are incomplete.

This is a powerful statement guaranteeing that if a globally hyperbolic cosmological spacetime is everywhere expanding at a finite rate it must have begun in a singular state a finite time ago. The same conclusion must be reached with respect to the future of a spacetime that is everywhere contracting. This theorem has two weaknesses. Firstly it requires global hyperbolicity and, secondly, it says nothing about the role played by inhomogeneities during gravitational collapse. We therefore review another singularity theorem, due to Penrose and Hawking[23], which remedies this:

⁹The proofs for all singularity theorems relevant for $(3+1)$ -dimensional physics can be found in refs. [23, 22]. Their generalization to $(n+1)$ -dimensional gravity is straightforward and implies that the theorems hold without modification for $n \neq 3$. The case $n = 1$ is special.

Theorem II: Suppose a spacetime $(\mathcal{M}, g_{\mu\nu})$ satisfies $R_{\mu\nu}v^\mu v^\nu \geq 0$ for all timelike and null vectors v^μ as well as the timelike and null generic energy conditions. Then if the spacetime is a closed cosmology or there exists a point p such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence, then $(\mathcal{M}, g_{\mu\nu})$ must contain at least one incomplete timelike or null geodesic.

The most important aspect of this theorem for us is that it eliminates the assumption that the spacetime is expanding everywhere on the spatial hypersurfaces. In principle this allows one to treat cases where collapsing geometries are inhomogeneous. The new ingredient in this formulation is that the generic timelike and null energy condition must be satisfied. This will be true if each timelike (null) geodesic in $(\mathcal{M}, g_{\mu\nu})$ possesses at least one point where $R_{\mu\nu\rho\lambda}\xi^\mu\xi^\lambda \neq 0$ ($R_{\mu\nu\rho\lambda}k^\mu k^\lambda \neq 0$). It would be surprising to find interesting cosmological spacetimes that do not satisfy these generic conditions. For example, for a homogeneous and isotropic FLRW cosmology the timelike condition implies that $\ddot{a} \neq 0$ somewhere in the spacetime.

3. Geometrical bounces and singularity resolution

One of our aim is to find manners by which cosmological singularities are resolved in the context of the effective theories obtained from string theory. One way this could happen is if the spacetime is allowed to get out of a phase of contraction by bouncing. It has long been known that in four-dimensional conventional Einstein gravity this is prohibited (see, *e.g.*, ref. [28]) unless the matter supporting the geometry can violate the SEC. In this section we review this argument in all generality and show that it holds for $n \neq 3$ as well. However we observe that time-dependent bounces should be allowed on lower-dimensional sub-manifolds when considering dimensional reductions of higher-dimensional theories for which the SEC is *not violated*. This can have interesting implications for cosmology. For example it could mean that a bounce has occurred in the form of a ‘non-singular big-bang’ in the past. Of course since the SEC holds for the higher-dimensional spacetime the singularity theorems presented in section 2.3 predict the existence of singular points. However these could in principle be pushed back arbitrarily far in the past to times for which no cosmological data is available. In essence this would imply that in order to describe theoretically the relevant part of the cosmological evolution we may not need to worry about quantum gravity.

Clearly all this phenomenon does is allow us to push the problem of dealing with quantum gravity (at a cosmological level) back in time. This is based on a general geometrical effect allowing for exotic phenomena to happen in (3+1) dimensions due to the compensating effect of the dynamics in the transverse dimensions. A related manifestation of this was recently considered when attempts were made to explain four-dimensional positive acceleration in the context of higher-dimensional supergravity theories (see, *e.g.*, refs. [29, 30, 31, 32, 33]). More concrete explanations and examples are provided in later sections.

3.1 Local dynamics of a bounce

We now study the physics of gravitational bounces by considering the local dynamics of the phenomenon. This simply means we are investigating a spacetime region where the general metric ansatz (2.1) can be written in the form (2.3) using Gaussian normal coordinates. The local analysis we present is complemented in section 3.3 by global considerations based on the cosmological singularity theorems introduced earlier.

We define a bouncing spacetime region as a co-moving volume element that goes through a minimum in finite time (the parameter n in this case). The volume element is defined by

$$V(\delta M) = \int_{\delta M} \sqrt{{}^{(n)}g} d^n x, \quad (3.1)$$

where δM denotes an open set of points on a spacelike hypersurface. It is also legitimate to regard $V(\delta M)$ as being part of a lower-dimensional manifold in a higher-dimensional spacetime. This will be relevant to the case where only some of the spatial dimensions are associated with a bounce. The integer n will then be understood to mean the number of bouncing spatial directions. The function $V(\delta M)$ will reach an extremum when its first order variation vanishes. This corresponds to the condition

$$\delta V(\delta M) = \int_{\delta M} \sqrt{{}^{(n)}g} (\text{Tr } K) \delta n(x) d^n x = 0. \quad (3.2)$$

Consequently a spacelike region associated with a vanishing trace for the extrinsic curvature corresponds to an extremum for the volume element. This will be a minimum (a bounce) when the second order variation of $V(\delta M)$ is positive, *i.e.*,

$$\delta^2 V(\delta M) = - \int_{\delta M} \sqrt{{}^{(n)}g} \left[(\text{Tr } K)^2 + \frac{\partial \text{Tr } K}{\partial n} \right] > 0. \quad (3.3)$$

In summary a spacelike region is said to bounce when $\text{Tr } K = 0$ and

$$\frac{\partial \text{Tr } K}{\partial n} < 0. \quad (3.4)$$

This conclusion is valid locally, *i.e.*, when the metric can be written like eq. (2.3) and for a small patch of a given hypersurface. This is easily generalized to the case where an entire hypersurface, obtained by integrating over all volume elements δM , is bouncing.

3.2 Solitary bounces and energy conditions

We have seen earlier that for gravity to be an attractive force the NEC and the SEC need to be satisfied. We also assume that the energy density is positive-definite (WEC) for the cases of physical interest considered here. These are characteristics we assume a higher-dimensional effective theory derived from a fundamental theory such as string theory must possess. However, as pointed out above, so-called anti-gravitational effects, *i.e.*, apparent violations of energy conditions, could happen in lower-dimensional theories obtained through compactification.

We consider a general spacetime \mathcal{M} composed of distinct spatial sub-manifolds

$$\mathcal{M} = \mathcal{R} \times M_1 \times \dots \times M_N, \quad (3.5)$$

where \mathcal{R} represents the timelike variable. For this special case we find the relations

$$\text{Tr } K = \sum_{a=1}^N \text{Tr } K_a, \quad \text{Tr } K^2 = \sum_{a=1}^N \text{Tr } (K_a)^2, \quad (3.6)$$

where K_a is the extrinsic curvature associated with the sub-manifold M_a . The requirement that the energy density be positive is equivalent to the inequality

$$\rho = \frac{1}{2} {}^{(n)}R + \frac{1}{2} [(\text{Tr } K)^2 - \text{Tr } K^2] \geq 0. \quad (3.7)$$

Clearly for a $N = 1$ spacetime to bounce (at which point $\text{Tr } K = 0$) the constraint (3.7) becomes

$${}^{(n)}R \geq \text{Tr } K^2. \quad (3.8)$$

For a $(n + 1)$ -dimensional FLRW cosmology this implies that the spatial curvature is positive because then $\text{Tr } K^2 = 0$ at the bounce. This is simply the well-known result that bounces can only happen for spacetimes foliated with spherical hypersurfaces.

Let us now consider the case $N > 1$ where only one sub-manifold, say M_N , goes through a bounce. Then, using eqs. (3.6) as well as assuming $\text{Tr } K_N = 0 = \text{Tr } K_N^2$, the positive energy condition becomes

$$^{(n)}R + \sum_{a=1}^{N-1} [(\text{Tr } K_a)^2 - \text{Tr } K_a^2] \geq 0. \quad (3.9)$$

It is clear that for many spacetimes the second term in eq. (3.9) can be positive at the bounce. For $N = 2$ this term is positive-definite when we consider a product spacetime of isotropic and homogeneous spaces (see section 3.4 for details). This geometric effect affords us much leeway in getting bounces on spatial sub-manifolds while maintaining positivity of energy everywhere. A similar conclusion is reached with respect to having the WEC satisfied everywhere.¹⁰ The SEC requires that $R_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ for any timelike vector, *i.e.*,

$$\frac{\partial \text{Tr } K}{\partial n} - \text{Tr } K^2 \geq 0. \quad (3.10)$$

For $N = 1$ this constraint excludes spacetimes where any portion of a n -dimensional hypersurface would bounce. In the case of a product spacetime of the form (3.5) if the sub-manifold M_N bounces, the SEC takes the form

$$\sum_{a=1}^{N-1} \frac{\partial \text{Tr } K_a}{\partial n} - \sum_{a=1}^{N-1} \text{Tr } (K_a)^2 - \left| \frac{\partial K_N}{\partial n} \right| \geq 0. \quad (3.11)$$

This last expression implies that for a bounce to occur on M_N the first term on the left-hand-side (LHS) must be positive and large enough to make the whole expression positive. The conclusion is basically that bounces on sub-manifolds are clearly not excluded even in theories where the energy conditions are satisfied. This last statement obviously applies to spacetimes where there is positive acceleration on M_N . However in this case one does not need to worry about the extra constraint that $\text{Tr } K_N$ must vanish.

3.3 The compulsory singularity

The general system we have in mind is $(n + 1)$ -dimensional Einstein gravity coupled to sources originating from the effective theories describing the low energy dynamics of string theories. We are also taking the point of view that these higher-dimensional theories respect the SEC. As shown above this prevents the occurrence of phenomena such as positive acceleration and bounces if the spacetime is of the form $\mathcal{R} \times M$,¹¹ *i.e.*, when it can be written locally in the form (2.3). Relevant cosmological backgrounds are such that, at least in some small interval δn , the trace of the intrinsic curvature does not vanish (*i.e.*, $\text{Tr } K \neq 0$ on a subset of Cauchy surfaces). According to Theorem I presented in section 2.3, this implies that singular points must develop either in the past or the future of this region (depending on the sign of $\text{Tr } K$) if the SEC is satisfied. It might be relevant to consider cases for which the sign and magnitude of $\text{Tr } K$ vary on a given constant n hypersurface.¹² In this case theorem II implies that at least one singular point will form either in the past or the future of the Cauchy surface under consideration. In other words we expect that inhomogeneities

¹⁰The WEC implies that $^{(n)}R + \text{Tr } K^2 - n(\text{Tr } K)^2 + (n - 1) \frac{\partial K}{\partial n} \geq 0$.

¹¹ M is not necessarily homogeneous and isotropic.

¹²This is important when studying the role of inhomogeneities.

of supergravity fields cannot prevent the appearance of singular points in the past of an expanding phase.¹³

We are mostly interested in geometries that can (at least locally) be written in the form of a product spacetime as in (3.5). Again any interesting cosmology will be associated with at least some Cauchy surfaces where the trace of the extrinsic curvature does not vanish, *i.e.*,

$$\text{Tr } K = \sum_{a=1}^N \text{Tr } K_a \neq 0 \quad (3.12)$$

in some finite interval δn . Following the reasoning of the previous paragraph this implies that singular points will develop either in the future or the past of this region. Supergravity theories typically respect the SEC¹⁴ which implies that no regular time-dependent solutions (in the sense of the cosmological singularity theorems) can be obtained in this context. Naively we would expect that solutions submitted to the constraint $\text{Tr } K = 0$ everywhere will lead to non-singular cosmologies because the singularity theorems are inapplicable. This is not the case since the SEC, which takes the form (3.10), then becomes

$$\text{Tr } K^2 \leq 0. \quad (3.13)$$

This is obviously impossible to satisfy for any non-trivial time-dependent geometry. This will be illustrated more clearly in section 3.4 when a more specific metric ansatz is considered.

The main conclusion here is that non-singular time-dependent solutions do not exist in supergravity. This is also true for spacetimes with phases of positive acceleration and gravitational bounces. As pointed out in section 2.2 exceptions exist since there are stringy sources violating the SEC. This could include spacetimes supported by a tachyon source associated with an unstable brane with spatial co-dimension one or zero. Such gravitational solutions were recently studied in ref. [34] in the context of bouncing cosmology. The other exception consists in considering cosmologies supported by a matter content which includes a space-filling anti-symmetric form-fields.¹⁵ Examples related to this are presented in the remaining sections of this paper.

3.4 Singular Hubble loops

It is always possible to obtain interesting phenomena such as positive acceleration and bounces on a sub-manifold in a higher-dimensional spacetime where the SEC is *not violated*. The latter condition is violated only effectively from the point of view say of an observer on the lower-dimensional manifold. This observation is motivated by the local analysis presented in section 3.2 (see ref. [29] for the case of positive acceleration). However while the spacetime might appear to be non-singular with respect to the effective metric on this sub-manifold, there will always be singular points in the higher-dimensional realization. This conclusion is reached by considering global aspects via the cosmological singularity theorems presented in section 2.3. Generally phases with positive acceleration or a gravitational bounce can be used as a mechanism to avoid the appearance of singular points. Our point is that this can occur, for instance, on a (3+1)-dimensional sub-manifold embedded in a higher-dimensional theory with sources not violating the SEC. This represents a mechanism for which the singularity resolution on the lower-dimensional spacetime is only an illusion.

Let us illustrate more concretely this type of behavior. We consider globally hyperbolic time-dependent geometries composed of N distinct homogeneous and isotropic spatial sub-manifolds

¹³This conclusion also applies to less conventional sources such as the $p \leq 7$ tachyon considered in refs. [16, 17].

¹⁴The very few exceptions to this rule are exploited in section 6.

¹⁵If the form field is the only matter component it essentially plays the role of a cosmological constant.

with different scale factors. The metric ansatz is

$$ds^2 = -dt^2 + \sum_{i=1}^N a_i(t)^2 d\Sigma_{n_i, k_i}^2, \quad (3.14)$$

where n_i is the dimensionality ($n = \sum_{i=1}^N n_i$) and k_i the spatial curvature associated with each sub-manifold. The intrinsic curvature of the Cauchy surfaces is given by

$${}^{(n)}R = \sum_{i=1}^N \frac{n_i(n_i - 1)k_i}{a_i^2}, \quad (3.15)$$

and the average expansion rate of infinitesimally nearby geodesics is

$$\theta = -\text{Tr } K = \sum_{i=1}^N n_i H_i, \quad (3.16)$$

where we introduced the Hubble factors $H_i = \dot{a}_i/a_i$. Another useful relation is

$$\text{Tr } K^2 = \sum_{i=1}^N n_i H_i^2, \quad (3.17)$$

which allows us to write down

$$(\text{Tr } K)^2 - \text{Tr } K^2 = \sum_{i=1}^N n_i(n_i - 1)H_i^2 + (\text{cross - terms}), \quad (3.18)$$

where the cross-terms can be either positive or negative depending on the relative sign of the H_i 's. Using eq. (3.7) the energy density becomes

$$\rho = \frac{1}{2} \sum_{i=1}^N n_i(n_i - 1) \left(H_i^2 + \frac{k_i}{a_i^2} \right) + (\text{cross - terms}), \quad (3.19)$$

which, for $N = 1$, gives the usual

$$\rho = \frac{n(n-1)}{2} \left(H^2 + \frac{k}{a^2} \right). \quad (3.20)$$

The SEC (3.10) is equivalent to the constraint

$$\sum_{i=1}^N n_i \frac{\ddot{a}_i}{a_i} \leq 0, \quad (3.21)$$

which cannot be satisfied for positive acceleration if $N = 1$.

We have established earlier that a cosmological spacetime cannot be associated with a gravitational bounce. However we have shown that a sub-manifold within a higher-dimensional spacetime can contain a bouncing phase. The drawback is that while this sub-space might appear regular, singular points will always develop as seen from the higher-dimensional point of view. In what follows we illustrate this generic behavior explicitly for the case $N = 2$.

The assumption we make is that the scale factor a_2 goes through a bounce ($\dot{a}_2 = 0$ and $\ddot{a}_2 > 0$). The expression (3.7) for the density of energy (assumed positive) at the lower-dimensional bounce becomes

$$\rho_c = \frac{n_2(n_2 - 1)k_2}{2a_2^2} + \frac{n_1(n_1 - 1)}{2} \left(H_1^2 + \frac{k_1}{a_1^2} \right). \quad (3.22)$$

The $N = 1$ constraint requiring $k = +1$ as a necessary condition for a bounce is relaxed for $N = 2$. Bounces can happen as long as the contribution to ρ from the transverse directions is sufficient to make the RHS of eq. (3.22) positive. In particular bounces can in principle occur even for flat ($k_2 = 0$) and hyperbolic ($k_2 = -1$) foliations.

The volume of the Cauchy surfaces is proportional to the function

$$V(t) = a(t)^n = a_1(t)^{n_1} a_2(t)^{n_2}, \quad (3.23)$$

where $a(t)$ is an average scale factor. The second derivative of this expression is

$$\ddot{V} = n a^n \left[\frac{\ddot{a}}{a} + (n-1) \left(\frac{\dot{a}}{a} \right)^2 \right]. \quad (3.24)$$

Using this last expression we find

$$n \frac{\ddot{a}}{a} = n_1 \frac{\ddot{a}_1}{a_1} + n_2 \frac{\ddot{a}_2}{a_2} - \frac{n_1 n_2}{n} [H_1 - H_2]^2. \quad (3.25)$$

The RHS of eq. (3.25) must be negative since the SEC is satisfied in the higher-dimensional theory. If a bounce occurs on the lower-dimensional manifold with scale factor a_2 , the constraint (3.21) becomes

$$n_1 \frac{\ddot{a}_1}{a_1} \leq - \left| n_2 \frac{\ddot{a}_2}{a_2} \right|. \quad (3.26)$$

This implies that during the lower-dimensional bounce the dynamics of the transverse dimensions must be such that \ddot{a}_1/a_1 is negative with a magnitude large enough to compensate for the positivity of the acceleration associated with a_2 . Of course the constraint $\ddot{a}/a \leq 0$ must also be satisfied.

To illustrate this we consider a pictorial approach. All $N = 2$ cosmological evolutions can be represented by a parametric curve in the (H_1, H_2) plane. Non-singular asymptotically flat (or, more precisely, asymptotically FLRW) solutions would be associated with trajectories asymptoting (*i.e.*, in the infinite past and future) to the point $(H_1 = 0, H_2 = 0)$. The manner by which these asymptotic regions are reached is crucial information with respect to the global features of the spacetime. In particular the ‘speed’ with which the attractor points are attained will be determined by the dominant matter component at early and late time. Consequently all non-singular cosmological evolutions should be represented by closed loops in the two-dimensional (H_1, H_2) plane (see, for example, curve I on figure 2).¹⁶ However we have seen that in the context of higher-dimensional gravitational theories respecting the SEC the allowed trajectories cannot be closed based on the application of cosmological singularity theorems. In order to obtain closed trajectories new ingredients violating the SEC should be introduced. In section 6 we provide examples of this for asymptotically de Sitter (dS) spacetimes. Of course in this case the location of the endpoints of the (H_1, H_2) trajectories is changed.

A more realistic trajectory is represented by curve II on figure 2. It corresponds to a geometry evolving out of a big-bang in the infinite past (when $H_1 \rightarrow +\infty$ and $H_2 \rightarrow 0$), passing through a phase where a_2 bounces ($H_2 = 0$ and H_1 is finite) and finally evolving toward the $(H_1 = 0, H_2 = 0)$ point in the infinite future.¹⁷ The effective gravity seen on the manifold with scale factor a_2 appears non-singular (*e.g.*, $\lim_{t \rightarrow \pm\infty} H_2 = 0$). However the Hubble factor H_1 blows up at $t = -\infty$ which

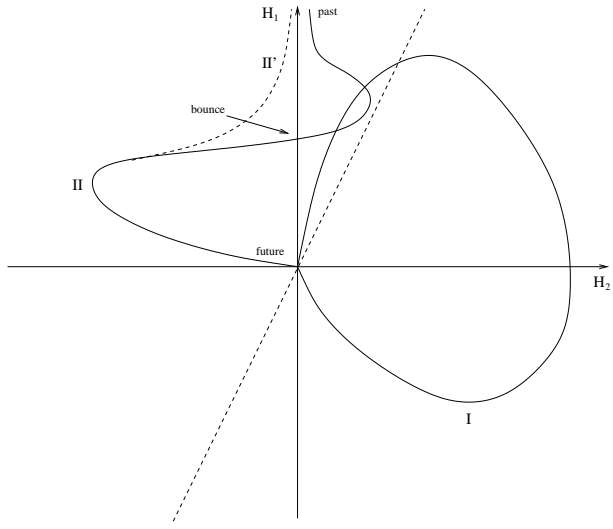


Figure 2: Typical trajectories in the (H_1, H_2) plane associated with $N = 2$ cosmological spacetimes. Curve I represents a non-singular evolution between asymptotic regions with $H_i \rightarrow 0$ for large $|t|$. Curve II shows a singular evolution including a bounce for the scale factor a_2 at intermediate time.

¹⁶The generalization to spacetimes with more sub-spaces is straightforward.

¹⁷The time-reversed evolution would correspond to a spacetime evolving into a big-crunch singularity.

suggests that there is a curvature singularity in the past. Such a spacetimes would be an example of the mirage singularity resolution described earlier.

Physically allowed trajectories on the (H_1, H_2) plane must satisfy other constraints such as positivity of the energy density, the SEC and the NEC. The line $n_1 H_1 + n_2 H_2 = 0$ (dashed line on figure 2) is special because it corresponds to an extremum of the volume function $V(t)$. Of course it cannot be traversed if the extremum corresponds to a minimum, *i.e.*, a bounce. A different example is depicted by curve II' on figure 2. In this case there is no bounce associated with a_2 and a curvature singularity is still present in the future. As for curve II the effective geometry associated with a_2 is non-singular. Because there is no bounce the corresponding effective spacetime will be regular and forever expanding. This situation is allowed because the SEC is effectively violated and the singularity theorems lose their predictive power.

4. Spacetimes with positive acceleration

Simple compactifications of supergravity theories lead to lower-dimensional effective actions where moduli fields such as the dilaton acquire potentials. These are typically of the form

$$V(\phi) = \Lambda e^{-\alpha\phi}, \quad (4.1)$$

where the scale Λ is set by the magnitude of fluxes and/or the internal curvature. In section 5 we consider explicit examples. The constant α is also determined by the ingredients present in the compactification scheme. We will see that its value is critical in the determination of whether or not non-singular cosmological solutions exist. Potentials of the form (4.1) are of interest to us because for $\Lambda > 0$ the corresponding scalar field can violate the SEC.

In this section we provide a study of spacetimes supported by homogeneous scalar fields with potentials of the form (4.1). In section 4.1 we begin by reviewing FLRW cosmology in $(m+1)$ -dimensions and explain some relevant aspects associated with spacetimes for which the SEC is violated. Then in section 4.2 we present analytic solutions for spacetimes with flat foliations and study their qualitative features. In section 4.3 we study the corresponding spacetimes with positive spatial curvature and, in particular, we consider geometries containing a bounce. All non-singular spacetimes we consider in this section are asymptotically FLRW with positive acceleration. In section 5 we will consider uplifting the solutions described in this section to solutions of ten- and eleven-dimensional supergravity.

4.1 Bounces and FLRW cosmology

Before proceeding with examples we make general remarks about conventional homogeneous and isotropic $(m+1)$ -dimensional FLRW cosmology. These comments are not novel in any way but will be useful in the remaining sections of this paper.

The invariance of gravitational actions under diffeomorphisms implies that the stress-energy tensor is covariantly conserved, *i.e.*, $\nabla^\mu T_{\mu\nu} = 0$. For a metric of the form

$$ds^2 = -dt^2 + a(t)^2 d\Sigma_{m,k}^2, \quad (4.2)$$

the continuity equation implies

$$\dot{\rho} = -m \frac{\dot{a}}{a} (\rho + p). \quad (4.3)$$

By inspection of eq. (4.3) we note that gravitational sources that are not respecting the WEC lead to pathologies. In fact if $\rho + p < 0$ an expanding universe is associated with an increasing density of energy. For $\rho = -p$ the energy density is constant which corresponds to dS space.

We assume that the sources obey the simple equation of state $p = w(t)\rho$. Solving eq. (4.3) we find

$$\ln \rho = -m \int dt (1+w) \frac{d \ln a}{dt}. \quad (4.4)$$

Clearly $w(t)$ is not known beforehand but it can be useful to assume it to be a constant. This can be an acceptable approximation when studying different phases of a given cosmology. The solutions to eq. (4.4) are then of the form

$$\rho = C a^{-m(1+w)}, \quad (4.5)$$

where C is a constant. Frequently encountered cases include pressureless non-relativistic dust ($\rho \sim a^{-m}$, $w = 0$), radiation ($\rho \sim a^{-(m+1)}$, $w = m^{-1}$) and a cosmological constant ($w = -1$). Other types of sources that are more stringy include, for example, the tachyon matter ($w = 0$) [26] and a gas of \tilde{p} -branes for which the equation of state is [35]

$$w = \frac{(\tilde{p} + 1)v^2 - \tilde{p}}{m + 1}, \quad (4.6)$$

where v is the magnitude of the average velocity associated with these spatially extended objects. It is interesting to note that a gas of \tilde{p} -branes violates the SEC for $\tilde{p} \geq 8$ as was found for conventional static and unstable $D\tilde{p}$ -branes in section 2.2.

In order to determine which component among the sources will dominate at different moments of the evolution it is useful to consider the ratio

$$\frac{\rho_{w_1}}{\rho_{w_2}} \sim \frac{1}{a^{m(w_1 - w_2)}}. \quad (4.7)$$

The integration constant in eq. (4.5) is important so studying such ratios gives us only a crude understanding of the system of interest. Clearly when $w_1 > w_2$ the component associated with w_2 dominates for large values of the scale factor (large spatial volumes) and, conversely, the component w_1 dominates for small spatial volumes. For example, the ratio for pressureless dust and radiation is $\rho_r/\rho_m \sim 1/a$, which leads to the well-known result that the late time (large a) evolution of the universe is matter-dominated. The density ratio of cosmological constant to any kind of matter with equation of state w is $\rho_\Lambda/\rho_w \sim a^{m(1+w)}$. For matter respecting the SEC it is clear that for early time dynamics (small a) a cosmological constant term tends to be overwhelmed. At late times (large a) however the cosmological constant always dominates.

The point of view here is that the $(m+1)$ -dimensional theory of gravity we are studying is derived from a higher-dimensional theory such as supergravity. This is why we consider a matter content which can in principle violate the SEC, *i.e.*, it can induce periods of positive acceleration. A certain combination of the field equations gives

$$\frac{\ddot{a}}{a} = -\frac{8\pi G_N}{m-1} [(m-2)\rho + mp]. \quad (4.8)$$

Clearly the sign of the RHS in eq. (4.8) determines whether the acceleration is positive or negative. As pointed out earlier the SEC requires that $(m-2)\rho + mp \geq 0$ which implies that sources satisfying this can only support negative acceleration. This also implies that isotropic and homogeneous $(m+1)$ -dimensional gravitational backgrounds sourced by matter respecting the SEC cannot bounce. This is clearly not the case if $-1 \leq w < (\frac{2}{n} - 1)$, *i.e.*, when w is such that the SEC is violated.¹⁸ This includes dS space which is in fact a bouncing spacetime when written in global coordinates.

The Friedmann constraint (3.20) can be written in the form

$$\frac{1}{2}\dot{a}^2 - \frac{8\pi G_N}{m(m-1)} (\rho a^2) = -\frac{k}{2}. \quad (4.9)$$

¹⁸The WEC is violated for $w < -1$.

This is equivalent to the first order equation governing the classical dynamics of a point particle if we replace a with the spatial displacement x . The conserved energy is then $-k/2$ and the potential function is given by

$$V(a) = -\frac{8\pi G_N}{m(m-1)} \left(e^{-m \int dt(1+w) \frac{d \ln a}{dt}} a^2 \right). \quad (4.10)$$

As pointed out above this is a useful analogy only if the equation of state is not time-dependent in which case we get

$$V(a) = -\frac{8\pi G_N}{m(m-1)} \frac{C}{a^{m(1+w)-2}}, \quad (4.11)$$

where $C > 0$ ($C < 0$) for positive (negative) energy density. For $k = 0$ the solution to eq. (4.9) is of the form $a \sim t^{\frac{2}{n(1+w)}}$ for $w \neq 1$ and for $w = -1$ we get the usual dS exponential.

As pointed out earlier we are interested in spacetimes that are bouncing or, at least, include phases of positive acceleration. Bouncing spacetimes have co-moving volumes evolving in such a way as to connect two different asymptotic vacua with large and possibly forever expanding spatial volumes. As is clear from the Friedmann constraint, matter sources satisfying the WEC can lead to a bounce only if the spatial curvature is positive ($k = +1$). The sources that support these gravitational backgrounds must therefore dominate the spatial curvature at late and early time ($t \rightarrow \pm\infty$) in order to prevent the apparition of cosmological singularities. This characteristic is also required of a realistic cosmological model if it is expected to conform with the observation suggesting the spatial curvature is currently very small [1]. The latter condition will be satisfied for realistic cosmological models predicting that

$$\frac{k/a^2}{8\pi G_N \rho} \quad (4.12)$$

is presently small. In principle this implies our universe could have negative spatial curvature as well.

For matter with a constant equation of state the SEC implies that the cosmological acceleration is negative. Curve I on figure 3 represents a typical potential $V(a)$ associated with matter respecting the SEC. Using the point particle analogy, the line labelled $E = -1/2$ (we consider the $k = +1$ case) represents the conserved energy. In this case the only $a(t)$ -trajectories which are kinematically allowed are those beginning their evolution for $a < a_c$.¹⁹ However they always lead to a turning point at $a = a_c$ followed by period of contraction leading to a singularity as predicted by Theorem I of section 2.3. Curve II on figure 3 is associated with matter violating the SEC ($-1 \leq w < -1 + 2/n$). Cosmologies corresponding to initial conditions fixed at some t_0 when $a > a_c$ will lead to eternal expansion with positive acceleration. A feature of the corresponding sources is that they can support a non-singular gravitational bounce. Beginning the evolution for large a the solutions can contract to a minimum scale a_c and then re-expand. We can consider other non-singular geometries supported by an equation of state which does not violate the SEC at

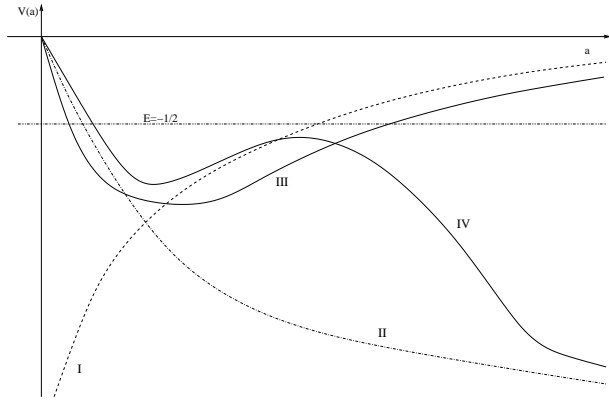


Figure 3: Schematic depiction of several hypothetical effective potentials $V(a)$ associated with different cosmological evolutions.

¹⁹Generically we refer to a_c as the point(s) where the curve $V(a)$ intersects the line $E = -1/2$.

all times. For example, curve IV can roughly be divided into three regions. For small a there is a positive acceleration region which is followed by a region where the SEC is satisfied. The transition from small to intermediate values of the scale factor corresponds to positive acceleration following by deceleration (negative acceleration) not unlike the transition between the inflationary and the radiation-dominated phases in the standard model. For large a the equation of state depicted on curve IV leads to a speed-up following the radiation→matter-dominated era. This could correspond to the current observed accelerating state of the universe [1]. We note that this picture is not inconsistent with a bounce for small values of the scale factor. This bounce could in principle be associated with the dynamics of the inflaton. Curve III is perhaps less relevant. There are then two critical points where the geometry can bounce. Given appropriate initial conditions, the corresponding spacetimes could be going through many cycles of collapse and re-expansion without developing singularities.

The conclusion of the over-simplified analysis performed in this section is nevertheless quite general. In order to obtain a homogeneous, isotropic and non-singular bouncing spacetime with positive spatial curvature there must be three regions of positive acceleration, *i.e.*, the past ($t \rightarrow -\infty$), the bouncing region and the future ($t \rightarrow +\infty$). A recent example of this can be found in ref. [34] where a tachyon-cosmological constant system is analyzed. Other examples of such systems are described in sections 4.3 and 6.

4.2 The infinite throat

In this section we consider flat ($k = 0$) solutions associated with a potential of the form (4.1). We consider why, although the sources can violate the SEC, the singularity theorems can be applied and used to explain the existence of a spacelike curvature singularity. Then in section 4.3 we find non-singular $k = +1$ solutions containing a gravitational bounce.

The $(p + 1)$ -dimensional action for the system under study is

$$\int d^{p+1}x \sqrt{-g} \left[R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \quad (4.13)$$

The equations of motion are found to be

$$R_{\mu\nu} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{p-1} g_{\mu\nu} V(\phi), \quad (4.14)$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \partial^\mu \phi) - \frac{\partial V}{\partial \phi} = 0, \quad (4.15)$$

which must be supplemented with a first order constraint (*e.g.*, $T_{tt} = G_{tt}$ for cosmological applications).

To facilitate the obtention of analytical solutions we consider a time-dependent homogeneous metric ansatz with a non-trivial lapse function,

$$ds^2 = -e^{2A(t)} dt^2 + e^{2B(t)} d\Sigma_{k,p}^2, \quad (4.16)$$

where the p -dimensional Euclidean metric $d\Sigma_{k,p}^2$ is

$$d\Sigma_{p,k}^2 = \frac{d\xi^2}{1 - k\xi^2} + \xi^2 d\Omega_{p-1}^2. \quad (4.17)$$

The spatial metric $d\Omega_{p-1}^2$ is that associated with a unit $(p - 1)$ -dimensional hypersphere. For $k = +1$ expression (4.17) is the unit metric on S^p . For $k = -1$ the unit metric is the hyperbolic p -dimensional space H^p . Using eq. (4.16) and assuming the geometry is supported by a homogeneous

scalar field, $\phi = \phi(t)$, the relevant equations of motion become

$$\ddot{B} + \dot{B} \left(p\dot{B} - \dot{A} \right) = \frac{1}{p-1} e^{2A} V(\phi) - (p-1) k e^{2(A-B)}, \quad (4.18)$$

$$\ddot{\phi} + \dot{\phi} \left(p\dot{B} - \dot{A} \right) + e^{2A} \frac{\partial V}{\partial \phi} = 0, \quad (4.19)$$

with the Friedmann constraint

$$\frac{p(p-1)}{2} \left(\dot{B}^2 + k e^{2(A-B)} \right) = \frac{1}{4} \dot{\phi}^2 + \frac{1}{2} e^{2A} V(\phi). \quad (4.20)$$

It is always possible to use a change of variables of the form $t = t'(t)$ allowing us to choose a convenient form for the lapse function. In what follows we therefore make the gauge choice $A = pB$ which simplifies the equation of motion for the scalar field. Let us then define the volume function

$$V(t) = e^{2pB}, \quad (4.21)$$

which is related to the actual constant-time volume of the spatial sections through

$$\int d^p x \sqrt{-g} = e^{2pB} \int \sqrt{{}^{(p)}g}, \quad (4.22)$$

where ${}^{(p)}g$ is the determinant associated with the Euclidean metric (4.17). The acceleration²⁰ associated with the volume of the spacetime is

$$\ddot{V}(t) = 2pV(t) \left(\ddot{B} + 2p\dot{B}^2 \right). \quad (4.23)$$

In the $A = pB$ gauge the dynamics of the gravi-metric field $B(t)$ is governed by the equation

$$\ddot{B} = -(p-1) k e^{2(p-1)B} + \frac{1}{p-1} e^{2pB} V(\phi). \quad (4.24)$$

It is clear that for $k = 0$ and $V(\phi) > 0$ the solutions have a volume with eternal positive acceleration. This is also true when considering hyperbolic foliations. Conversely for $k = +1$ the curvature term contributes in such a way as to favor negative acceleration. Analytic solutions can be obtained for $k = 0$ (the case $k = +1$ is treated in section 4.3) and a potential of the form (4.1). The equations of motion are then

$$\ddot{B} - \frac{\Lambda}{p-1} e^{2pB-\alpha\phi} = 0, \quad (4.25)$$

$$\ddot{\phi} - \alpha \Lambda e^{2pB-\alpha\phi} = 0, \quad (4.26)$$

and the Friedmann constraint takes the form

$$p(p-1) \dot{B}^2 = \frac{1}{2} \dot{\phi}^2 + \Lambda e^{2pB-\alpha\phi}. \quad (4.27)$$

It is then straightforward to write down the solution for the scalar field in terms of the field $B(t)$,

$$\phi(t) = \alpha(p-1)B(t) + c_1 t + c_2, \quad (4.28)$$

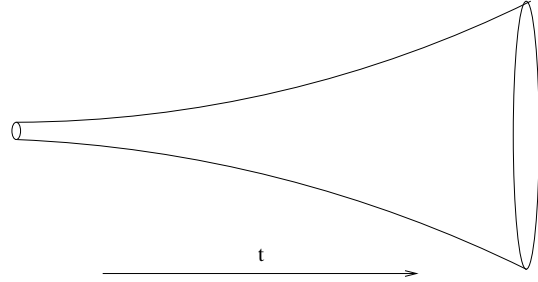


Figure 4: Schematic depiction of the infinite cosmological throat for $s = -1$. The volume function $V(t)$ vanishes as $t \rightarrow -\infty$ which corresponds to a singular spacelike horizon.

²⁰It is crucial to realize that this is not the acceleration measured using the so-called cosmological time. This will be clarified later in this subsection.

where c_1 and c_2 are at this stage undetermined constant parameters. Then we use the change of variables

$$h(t) = -\chi B(t) - \alpha (c_1 t + c_2) \quad (4.29)$$

to write down eq. (4.25) in the form

$$\ddot{h} + \frac{\chi\Lambda}{p-1} e^h = 0, \quad (4.30)$$

where $\chi = \alpha^2(p-1) - 2p$. For $\chi > 0$, the solution to the latter equation is

$$h(t) = \ln \left(\frac{p-1}{2\chi\Lambda c^2} \right) - \ln \left(\cosh^2 \left(\frac{t-t_0}{2c} \right) \right), \quad (4.31)$$

where t_0 and c are constants of integration. For $\chi < 0$ the solution becomes

$$h(t) = \ln \left(\frac{p-1}{2\chi\Lambda c^2} \right) - \ln \left(\cos^2 \left(\frac{t-t_0}{2c} \right) \right), \quad (4.32)$$

which develops curvature singularities in finite time at $|t| = c\pi + t_0$. The reason for this is clear since for $\chi < 0$ the second derivative of $h(t)$ is necessarily positive which implies that $\ddot{B} < 0$. This is inconsistent with the equations of motion.

We consider further the solutions for which $\chi > 0$, *i.e.*,

$$\alpha > +\sqrt{\frac{2p}{p-1}}, \quad (4.33)$$

where we chose the positive root because of its relevance for string compactifications. The features associated with $\alpha < 0$ time-dependent backgrounds are unchanged. To complete our analysis we need to make sure the solutions found are consistent with the Friedmann constraint (4.27). This leads to a relation between the integration constants c and c_1 ,

$$c^2 c_1^2 = \frac{p-1}{2p}, \quad (4.34)$$

or

$$c_1 = \frac{s}{c} \sqrt{\frac{p-1}{2p}}, \quad (4.35)$$

where s can either be $+1$ or -1 . As will be shown below this sign is important because it determines whether the spacetime is expanding or contracting. Finally the solution takes the form

$$B(t) = -\frac{1}{\chi} \left[h(t) + s \sqrt{1 + \frac{\chi}{2p}} \left(\frac{t}{c} \right) + \sqrt{\frac{\chi+2p}{p-1}} c_2 \right], \quad (4.36)$$

and the scalar field is given by

$$\phi(t) = -\sqrt{\frac{2p(p-1)}{\chi}} h(t) + s \sqrt{p-1} \left(\frac{1}{\sqrt{2p}} - \sqrt{\frac{\chi+2p}{\chi}} \right) \left(\frac{t}{c} \right) + c_2 \left(1 - \sqrt{\frac{2p(\chi+2p)}{\chi}} \right). \quad (4.37)$$

The constant t_0 can be removed by the field redefinition $t \rightarrow t - t_0$. There are therefore two physical parameters characterizing the cosmological spacetimes, *i.e.*, the constants c and c_2 .

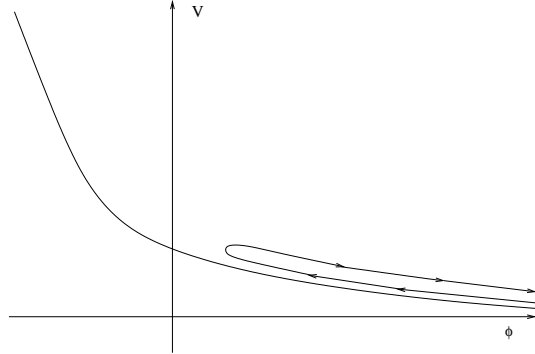


Figure 5: Typical scalar field trajectory associated with a bouncing cosmology. The potential is of the form $V(\phi) = \Lambda e^{-\alpha\phi}$ where $\alpha > 0$.

We now consider several important features associated with the spacetime solutions found. First let us determine the general behavior of the volume function $V(t)$. For $s = -1$ we find

$$\lim_{t \rightarrow \pm\infty} \ln V(t) = 2\alpha_{\pm} \frac{t}{c}, \quad (4.38)$$

where

$$\alpha_{\pm} = \frac{1}{\chi} \left(\pm 1 + \sqrt{1 + \frac{\chi}{2p}} \right) > 0. \quad (4.39)$$

The volume function is therefore such that it evolves from zero at $t = -\infty$ up to large values as $t \rightarrow +\infty$. This is represented on figure 4. The fact that $V(t)$ vanishes as $t \rightarrow -\infty$ implies that this region corresponds to an horizon. For $s = +1$ the spacetime is contracting instead and the horizon is at $t = +\infty$. The relevant curvature invariants are of the form

$$R = \frac{2p}{V(t)} f_1(\dot{B}, \ddot{B}), \quad (4.40)$$

$$R_{\mu\nu} R^{\mu\nu} = \left(\frac{2p}{V(t)} \right)^2 f_2(\dot{B}, \ddot{B}), \quad (4.41)$$

$$R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} = \left(\frac{2p}{V(t)} \right)^2 f_3(\dot{B}, \ddot{B}), \quad (4.42)$$

with the square of the Weyl tensor obviously vanishing at all times. The functions f_1 , f_2 and f_3 (polynomials in \dot{B} and \ddot{B}) are finite for all values of the parameter t . Therefore using eq. (4.38) we find there is a curvature singularity for $t \rightarrow -\infty$ (big-bang)²¹ but that the curvature vanishes as $t \rightarrow +\infty$.

The presence of singular points could not have been predicted using singularity theorems because the SEC can be violated in this case. For the solutions of interest the energy condition ($R_{tt} \geq 0$) is equivalent to the inequality

$$-\frac{1}{\cosh^2\left(\frac{t-t_0}{2c}\right)} + \frac{2(p-1)}{\chi} \left(\tanh\left(\frac{t-t_0}{2c}\right) + \sqrt{1 + \frac{\chi}{2p}} \right)^2 \geq 0. \quad (4.43)$$

We are able to show that this is always violated at least in some finite timelike interval.²² In the asymptotic regions we have

$$\lim_{t \rightarrow \pm\infty} R_{tt} = -\frac{2p}{\chi c^2} e^{\mp \frac{t}{c}} + \left(\pm 1 + \sqrt{1 + \frac{\chi}{2p}} \right)^2. \quad (4.44)$$

Since the first term on the RHS asymptotically vanishes this shows that the SEC is respected close to the horizon and also as $t \rightarrow +\infty$. In fact the large $|t|$ behavior of the metric is found to be

$$ds^2 = -d\tau^2 + \left(\frac{p\alpha_{\pm}}{c} \right)^{\frac{2}{p}} \tau^{\frac{2}{p}} d\mathbf{x}^2, \quad (4.45)$$

where we have used the change of variable

$$t = \frac{c}{p\alpha_{\pm}} \ln \left(\frac{p\alpha_{\pm}}{c} \tau \right). \quad (4.46)$$

The asymptotic behavior (4.45) corresponds to the asymptotic equation of state $w = 1$. This explains why a singularity appears at $t = -\infty$. Bounces are not allowed in this spacetime so in the

²¹For $s = +1$ there is a big-crunch at $t = +\infty$.

²²This leads to a period of positive cosmological acceleration as was considered in refs. [29, 31].

past the spacetime is expanding in a region where the SEC is satisfied. Theorem I therefore applies and predicts that singular points will develop (in this case a genuine curvature singularity at $\tau = 0$ or $t = -\infty$). We have shown that R_{tt} extrapolates between two positive values between $t = -\infty$ and $t = +\infty$. The question is whether there are points in between where $R_{tt} < 0$. The LHS of the inequality (4.43) is minimized for

$$\frac{t_c}{2c} = \operatorname{arctanh} \left(-\frac{1}{\sqrt{1 + \frac{\chi}{2p}}} \right). \quad (4.47)$$

Plugging this back in R_{tt} we find that it is always negative for $t = t_c$. This implies that the $k = 0$ spacetimes found are always associated with an intermediate period of positive acceleration. The duration of this phase can be obtained by studying further the inequality (4.43).

4.3 Bouncing spacetimes

We have shown that the $k = 0$ time-dependent solutions with a potential of the form (4.1) are always singular. In this subsection we consider the corresponding $k = +1$ spacetimes. Our analysis is based on the FLRW metric ansatz

$$ds^2 = -dt^2 + a(t)^2 d\Omega_p^2. \quad (4.48)$$

The equations of motion are

$$\frac{\ddot{a}}{a} = -\frac{1}{2p}\dot{\phi}^2 + \frac{\Lambda}{p(p-1)}e^{-\alpha\phi}, \quad (4.49)$$

$$\ddot{\phi} = -p\frac{\dot{a}}{a}\dot{\phi} + \alpha\Lambda e^{-\alpha\phi}, \quad (4.50)$$

with the Friedmann constraint

$$p(p-1) \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{1}{a^2} \right] = \frac{1}{2}\dot{\phi}^2 + \Lambda e^{-\alpha\phi}. \quad (4.51)$$

In particular we investigate cosmological solutions containing a gravitational bounce. Our results are obtained by solving the system of differential equations numerically. The strategy is to exploit the fact that both the equations of motion and the boundary conditions are time-reversal symmetric. The latter are chosen in order that the bounce occurs at $t = 0$,

$$\dot{a}(0) = 0 = \dot{\phi}(0), \quad \phi(0) = \phi_0. \quad (4.52)$$

The first order constraint imposes that the size of the $t = 0$ Cauchy surface is fixed by

$$a(0) = \sqrt{\frac{p(p-1)}{\Lambda e^{-\alpha\phi_0}}}. \quad (4.53)$$

The type of scalar field trajectories leading to a bouncing spacetime is depicted on figure 5. The scalar rolls from $\phi = +\infty$, reaches a maximum at $\phi = \phi_0$ and then rolls down again toward small values of the potential. The bounce of the spatial sections occurs precisely at the turning point for the scalar field. We solved the equations numerically to find the geometry corresponding to the roll down from $\phi = \phi_0$ at $t = 0$ to $\phi = +\infty$ at $t = +\infty$. The other half of the solution, *i.e.*, the past, is simply the time-reversed version of the $t > 0$ solution.

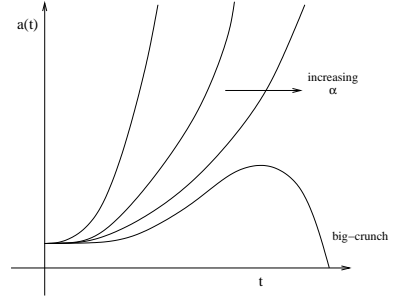


Figure 6: Schematic depiction of the scale factor behavior as a function the slope parameter α .

There are two important parameters in the system with boundary conditions (4.52). The combination $\Lambda e^{-\alpha\phi_0}$ characterizes the height of the potential when the scalar field is released from rest at $t = 0$. However the most relevant parameter (in terms of whether or not non-singular solutions exist) is the dimensionless quantity

$$\alpha = \left| \frac{\partial \ln V(\phi)}{\partial \phi} \right|, \quad (4.54)$$

which determines the slope of the potential. Without loss of generality we fixed $\Lambda = 1$ and varied ϕ_0 when scanning through all possible solutions. We were interested in determining what values of $\Lambda e^{-\alpha\phi_0}$ and α lead to non-singular evolutions. Secondly, we wanted to study the nature of the cosmological acceleration associated with the spatial sections.

It is relatively easy to generate solutions with a bounce. However most of those develop singular points while the scalar field rolls down the potential. We find there is a critical value $\alpha = \alpha_c$ above which the bouncing spacetimes always develop a curvature singularity. This feature is independent of what values of ϕ_0 and Λ are chosen. This is illustrated on figure 6. For $\alpha < \alpha_c$ we obtain non-singular bouncing geometries associated with asymptotic phases (large $|t|$) having positive acceleration. Smaller values of α lead to larger values of asymptotic acceleration as shown on figure 6. The critical value for $p = 3, 4, 5, 6, 7, 8, 9$ corresponds respectively to roughly $10\alpha_c = 6, 5, 4, 3, 3, 2, 2$.

Inspecting eq. (4.49) we see that the contribution of the scalar field to the acceleration is negative-definite while the form field contributes a positive-definite term.²³ Which contribution ultimately dominates determines the fate of the spacetime for large $|t|$. Let us consider the $t > 0$ case. Typically what happens for $\alpha > \alpha_c$ is that the kinetic term becomes dominant and drives the spacetime into a phase of contraction. Then since the SEC holds Theorem I from section 2.3 is applicable which supports our finding that singular points appear in the future. This depressing state of affair does not persist for $\alpha < \alpha_c$. In this case the contribution of the potential to the acceleration dominates asymptotically and the spacetime never enters a phase of contraction. This implies that the SEC is violated for large $|t|$, *i.e.*, the scale factor will behave like

$$\lim_{t \rightarrow \pm\infty} a(t) = t^m, \quad (4.55)$$

where $m > 1$. Among other things this implies that while \ddot{a}/a and \dot{a}/a vanish asymptotically, quantities such as \dot{a} are unbounded.²⁴ In fact the effective potential $V(a)$ (see section 4.1) will be of the form of either curve II or curve IV on figure 3.

We have determined that the non-singular $k = +1$ solutions have three phases of positive acceleration: the past, the bounce and the future. The qualitative behavior for the intermediary phases can take three different forms. Curve a) on figure 7 represents a common signature where the acceleration remains positive during the whole evolution. In some instances \ddot{a}/a always remains positive but develops a kink in finite time as represented by curve b). Interestingly, for some values of the parameters this kink drops below zero which corresponds to the spacetime entering a regime of negative acceleration. For example this happens for the case $p = 4$, $\alpha = 2/5$, $\Lambda = 1$ and $\phi_0 = 10$. The corresponding acceleration is depicted by curve c).²⁵ This behavior is interesting since it

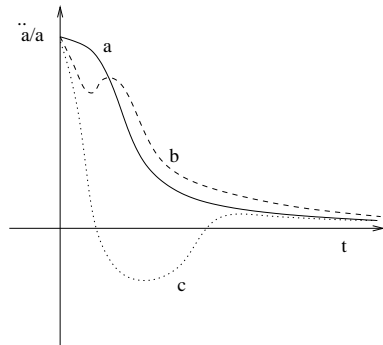


Figure 7: Schematic representation of the cosmological acceleration associated with some $k = +1$ solutions supported by a scalar field in a positive exponential potential.

²³This can of course be traced back to the fact that space-filling form fields violate the SEC.

²⁴This is of course not worrisome since \dot{a} is not an observable.

²⁵The form of the associated effective potential would be that represented by curve IV on figure 3.

corresponds to a phase of large positive acceleration (close to $t = 0$) followed by a phase of negative acceleration with a future characterized by a small (compared to that at $t = 0$) positive acceleration. The large $|t|$ contribution of the spatial curvature is negligible with respect to the energy density ($a^{-2}/\rho \simeq 0$). This is reminiscent of our own universe which begins with an inflationary phase followed by radiation– then matter–dominated era. The observed late time behavior is that of an accelerating spacetime with equation of state close to $w = -1$ [1].

5. Supergravity applications

In the previous section we found time–dependent gravitational solutions with periods of positive acceleration. The $k = 0$ solutions are singular while some of the $k = +1$ solutions are regular because of the presence of a bounce. In this section we consider whether or not these geometries can be embedded in ten–dimensional (more precisely Type IIA and Type IIB) and eleven–dimensional supergravity theories. However the resulting higher–dimensional geometries will always be singular. This is expected because all time–dependent solutions of ten– and eleven–dimensional supergravity contain singular points since the gravitational sources do not violate the SEC.²⁶

Suppose the regular $(p+1)$ –dimensional bouncing solutions found in section 4 can be embedded in a supergravity theory. Then the non–singular character in $(p + 1)$ dimensions is only an illusion of the compactification scheme. In fact based on Theorem I in section 2.3 the uplifted geometry must contain singular points associated with, for example, a breathing mode driving the higher–dimensional spacetime toward gravitational collapse (small spacetime volume) in a region where the lower–dimensional scale factor increases. In other words, in the past and/or the future of a lower–dimensional bounce, the volume of the full spacetime will be driven toward gravitational collapse although the four–dimensional geometry appears to be non–singular.

The idea that such exotic effects as positive acceleration and gravitational bounces are geometrical effects in gravitational theories with more than four dimensions can have far–reaching consequences for conventional cosmology. For example this geometrical effect could change our perspective on issues related to the hypothetical initial singular state sometimes associated with the big–bang. An interesting hypothesis is that rather than expanding out of a singular state our universe simply bounced off after having reached a finite size and expanded toward its current state. As shown in section 3 this is only physically realizable in theories with sources violating the SEC. The geometrical effect described above provides us with enough leeway to conjecture that bounces are not excluded in higher–dimensional models respecting the SEC. However since the singularity theorems predict the apparition of singular points the regular nature of the lower–dimensional spacetime is an illusion. This is still useful however because in principle the singular region can be made to appear arbitrarily far in the past of the bouncing region. For a lower–dimensional observer this would appear as though the big–bang singularity has been resolved by a bounce.

5.1 Flux compactifications

In this subsection we consider $d = (m + 1) = (p + 1 + n)$ –dimensional spacetimes of the form

$$ds^2 = G_{IJ}dx^I dx^J = e^{\xi\psi} g_{\mu\nu} dx^\mu dx^\nu + e^{2\psi} d\Sigma_{n,\sigma}^2, \quad (5.1)$$

where $g_{\mu\nu}$ is the metric associated with a $(p + 1)$ –dimensional Lorentzian spacetime and $I, J = 0, \dots, m$. The scalar field ψ can be regarded as a breathing mode for the maximally symmetric

²⁶As pointed out earlier there are a few exceptions. In section 6 we consider systems associated to those.

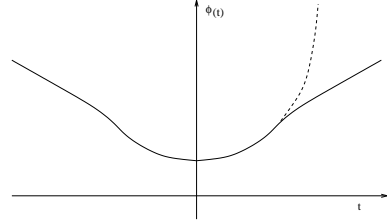


Figure 8: Typical evolution of a scalar field supporting a bouncing spacetime with $k = +1$. The dashed curve represents an evolution leading to a curvature singularity.

Euclidean manifold with curvature $\sigma = -1, 0$ or $+1$. The study of models with a transverse space associated with richer symmetry groups is beyond the scope of our work (see ref. [32]). The theories of interest to us are the ten- and eleven-dimensional supergravities so we consider a general Einstein frame action of the form

$$S = \frac{1}{16\pi G_{(m+1)}} \int d^{m+1}x \sqrt{-G} \left[{}^{(m+1)}R - \frac{1}{2} \partial_I \phi \partial^I \phi - \frac{1}{2(p+1)!} e^{a\phi} F_{[p+1]}^2 \right], \quad (5.2)$$

where $G_{(m+1)}$ is the d -dimensional Newton constant, ϕ is the dilaton field (absent for $m = 10$), and $F_{[p+1]}$ is the field strength associated with the Ramond–Ramond (RR) form fields C^p . In ten-dimensional supergravity the dilaton coupling is $a = (4-p)/2^{27}$ and our notation is $F_{[p+1]}^2 = F_{\mu_1 \dots \mu_p} F^{\mu_1 \dots \mu_p}$.²⁸

We now write down, starting with expression (5.2), an effective action for gravity on the $(p+1)$ -dimensional Lorentzian sub-manifold with metric $g_{\mu\nu}$. Before proceeding with the dimensional reduction we solve the equation of motion associated with the form field. For the ansatz $F_{\mu\mu_1 \dots \mu_p} = \epsilon_{\mu\mu_1 \dots \mu_p} A(x^\nu)$ the equation of motion

$$\partial_I \left(\sqrt{-G} e^{a\phi} F^{II_1 \dots I_p} \right) = 0, \quad (5.3)$$

is solved for

$$A(x^\nu) = C e^{-n\psi - a\phi}, \quad (5.4)$$

where C is a constant. Using this result and the conventional Kaluza–Klein ansatz (see ref. [36] for a modern treatment) we find the dimensionally reduced action

$$S = \frac{1}{16\pi G_N} \int d^{p+1}x \sqrt{-g} \left[{}^{(p+1)}R - \frac{n(n+p-1)}{p-1} \partial^\mu \psi \partial_\mu \psi - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\psi, \phi) \right], \quad (5.5)$$

where the effective potential for the dilaton and the breathing mode is given by

$$V(\psi, \phi) = \frac{C^2}{2} e^{-(a\phi + \frac{2np}{p-1}\psi)} - {}^{(n)}R e^{-2(1 + \frac{n}{p-1})\psi}. \quad (5.6)$$

In order to obtain a conventionally normalized Ricci term we used $\xi = -2n/(p-1)$ and the relation

$$\frac{1}{16\pi G_N} = \frac{1}{16\pi G_{(m+1)}} \int d^m y \sqrt{{}^{(n)}g}, \quad (5.7)$$

relating the $(p+1)$ -dimensional Newton constant (G_N) to the higher-dimensional gravitational scale. The kinetic term for the breathing mode in eq. (5.5) is not canonically normalized. This is fixed by applying

$$\psi \rightarrow \sqrt{\frac{p-1}{2n(n+p-1)}} \bar{\psi} \quad (5.8)$$

which changes the form of the potential to

$$V(\bar{\psi}, \phi) = \frac{C^2}{2} e^{-\left(a\phi + \frac{\sqrt{2np}}{\sqrt{(p-1)(n+p-1)}} \bar{\psi}\right)} - {}^{(n)}R e^{-2\left(1 + \frac{n}{p-1}\right) \sqrt{\frac{p-1}{2n(n+p-1)}} \bar{\psi}}. \quad (5.9)$$

²⁷This ensures that the dilaton does not couple to the RR fields kinetic term in the string frame.

²⁸We consider only form fields having their indices along p spatial directions on the Lorentzian manifold.

5.2 Effective potential and scalar field dynamics

The form of the exponential potential term for the dilaton in eq. (5.9) is determined by the magnitude and sign of a . If a is negative ($p > 4$ in Type IIA and Type IIB supergravity) then the solutions will evolve between two asymptotic regions where the dilaton contribution to the potential asymptotically vanishes, *i.e.*, as $\phi \rightarrow -\infty$ when $t \rightarrow \pm\infty$. This implies that the geometry extrapolates between two regions where the string coupling, $g_s = e^{2\phi}$, is small. If a is positive ($p < 4$ in supergravity) the corresponding solutions will include asymptotic regions where the string coupling is unbounded.

The spatial volume of the Cauchy surfaces associated with the metric (5.1) is controlled by the quantity

$$\int d^m x \sqrt{-G} = \int d^p x d^n y \sqrt{{}^{(n)}g} \left[\sqrt{-g} e^{-\frac{2n}{p-1}\bar{\psi}} \right], \quad (5.10)$$

where the time-dependent contributions are inside the square-brackets. Inspection of eq. (5.9) shows that the argument of the exponential functions in $\bar{\psi}$ is negative (for positive $\bar{\psi}$). Typically this implies that all (potentially non-singular) time-dependent solution are such that

$$\lim_{t \rightarrow \pm\infty} \psi(t) = +\infty. \quad (5.11)$$

This condition is physical since, for example, if it is not satisfied the potential energy of the scalar could become unbounded. This means, as suggested by eq. (5.10), that the contribution of the breathing mode to the evolution of the Cauchy surfaces favors a contraction to small size both in the asymptotic past and future.

Of course this last comment does not take into consideration the dynamics associated with the effective $(p+1)$ -dimensional cosmology (*i.e.*, the $\sqrt{-g}$ contribution to expression (5.10)). The time-dependence of $g_{\mu\nu}$ can, in principle, be non-singular and, for example, include a gravitational bounce. Of course this type of behavior is in principle allowed because the SEC is effectively violated in the lower-dimensional theory. However the higher-dimensional theory satisfies the latter condition which implies the presence of singular points. This strongly suggest that the breathing mode will always drive the full spacetime toward catastrophic gravitational collapse either (or both) in the past and the future.

5.3 The arduous ascension

We study whether or not the solutions found in section 4 can be embedded in Type IIA and Type IIB supergravity. These solutions correspond to truncations of the theories with the action (5.2) such that a single exponential term survives in the effective potential eq. (5.9). An example consists in considering the transverse space to be a n -dimensional torus (${}^{(n)}R = 0$). We also assume for simplicity that the dilaton is turned off and that $a = 0$. A generalization including the dilaton can be found in appendix A.

We begin by embedding the $k = 0$ solutions found in section 4.2. The solutions that can be consistently uplifted are those for which

$$\alpha = \frac{\sqrt{2np}}{\sqrt{(p-1)(n+p-1)}}, \quad \Lambda = \frac{C^2}{2}. \quad (5.12)$$

We consider only the expanding infinite throat ($s = -1$) which is associated, from the lower-dimensional perspective, to a singular horizon at $t = -\infty$. The important result is that the asymptotic volume of the uplifted solutions behaves like

$$\lim_{t \rightarrow \pm\infty} \ln \left[\sqrt{-g} e^{-\frac{2n}{p-1}\bar{\psi}} \right] = \kappa_{\pm} \frac{t}{c} = \left(2\alpha_{\pm}(p - \alpha n) + n \sqrt{\frac{2}{p(p-1)}} \right) \frac{t}{c}. \quad (5.13)$$

The sign of the constants κ_+ and κ_- is the determining factor here. We find, for $d = 10$ and $d = 11$, that κ_- is positive for all relevant values of p . This implies that the point $t = -\infty$ is also singular in the uplifted geometry. Now for $d = 10$ and for $d = 11$ we find that κ_+ becomes negative respectively for $p \geq 6$ and $p \geq 7$. The uplifted geometries for which $\kappa_+ > 0$ are expanding and non-singular in the future while those with $\kappa_+ < 0$ enter a phase of contraction in the future. This implies that singular points will appear as predicted by the cosmological singularity theorems. We have also verified explicitly that curvature singularities appear in the future of the $\kappa_+ > 0$ solutions. The resulting uplifted geometries are therefore associated with both a big-bang and a big-crunch singularity. This is an example where the non-singular nature of a lower-dimensional spacetime (the future of the effective $(p+1)$ -dimensional spacetime) is only an illusion of the compactification scheme considered.

The compactifications considered in this section do not lead to effective theories allowing us to embed the non-singular bouncing solutions found in section 4.3. It is not excluded however that those could be embedded for compactifications on manifolds associated with more interesting symmetries. The singular $(p+1)$ -dimensional bouncing solutions can easily be embedded in ten- or eleven-dimensional supergravity. It is conceivable that one of the two singularities (either the big-bang or the big-crunch) is actually lifted and disappears from the higher-dimensional point of view.

It would be very interesting to study further the $(p+1)$ -dimensional gravitational system associated with a potential (5.9) for which ${}^{(n)}R > 0$ and $C \neq 0$. In this case the curvature term will favor negative acceleration. For the $k = +1$ solutions this is a dangerous contribution if it comes to dominate over the SEC violating contributions during a phase of expansion. The dynamics of this system is the result of a constant competition between the curvature and the flux terms. If non-singular bouncing solutions are found the $t > 0$ region could correspond to a phase of negative acceleration between two phases of positive acceleration. It would be interesting to see if the phase of positive acceleration around $t = 0$ can be used as a realistic model of inflation. Then it would be interesting to verify whether the asymptotic region of positive acceleration can resemble closely enough the phase of positive acceleration that is currently observed in our universe [1].

6. Spacelike branes

We conclude this paper by considering an enigmatic class of time-dependent solutions in string theory: spacelike branes. The s-branes were conjectured to be the phenomenon associated with the creation of a D-brane from a closed string vacuum and its subsequent decay into closed strings [13]. These objects were studied from different perspectives. From a classical gravitational point of view, s-branes should correspond to non-singular time-dependent backgrounds extrapolating (in time) between two asymptotically FLRW spacetimes. Our main results here are twofold. First we close a gap left opened in the literature [37, 38] by showing that the space-filling s8-brane is singular. Then, perhaps more importantly, we propose a natural mechanism to resolve the singularities associated with the supergravity s-branes [14, 15, 17, 37]. The resulting non-singular s-brane configurations would be asymptotically dS.

The conventional approach to studying $s(p-1)$ -branes in the context of type IIA,B supergravity [13, 14, 15] consists in considering a geometry of the form

$$ds^2 = -dt^2 + a(t)^2 d\Sigma_{p,k_1}^2 + R(t)^2 d\Sigma_{9-p,k_2}^2, \quad (6.1)$$

coupled to homogeneous time-dependent dilaton and RR form field C^p . The symmetry group for the s-brane was argued in ref. [13] to be that associated with $k_1 = 0$ and $k_2 = -1$ but we do not adopt this convention in the following analysis and consider, for instance, spherical branes as well.

The Euclidean sub-manifold with scale factor $a(t)$ is the worldvolume of the unstable brane. As pointed out earlier any excitation of the sources in this problem, *i.e.*, the dilaton and the RR field, is always such that the SEC is satisfied. This implies that the supergravity solutions for s-branes are always singular [37] since the cosmological theorems reviewed in section 2.3 are applicable. However, as pointed out in refs. [37, 38] there is one exception: the s8-brane.

Recently so-called non-singular s-brane solutions were presented in the literature by considering some analytic continuations of known static black hole solutions [18] (see also refs. [39, 40]). In upcoming work [41] we consider in what sense these solutions evade the higher-dimensional singularity theorems presented in section 2.3.

6.1 The s8-brane and the rolling tachyon

The space-filling s8-brane is different from the other spacelike branes if only for the fact that it has no transverse spatial directions. If space-filling unstable branes are physical objects then the coupling of the associated open string tachyon to the RR field is of the form

$$S_{wz} = \int f(T) dT \wedge C^9, \quad (6.2)$$

where the form of $f(T)$ can be found in, *e.g.*, ref. [42]. The non-propagating form field C^9 is for example present in massive Type IIA supergravity (see ref. [43]).

As pointed out earlier in this work spacetime filling form fields (associated with a field strength with as many indices as there are spacetime dimensions) violate the SEC. This is interesting because it suggests that the s8-brane might be non-singular since the cosmological singularity theorems are not applicable in this case. We consider both the flat ($k = 0$) and the spherical ($k = 1$) space-filling branes. For $k = 1$ the solutions we consider should correspond to a ten-dimensional bouncing spacetime extrapolating between two asymptotically FLRW regions. The $k = 0$ solutions cannot go through a bouncing phase because, according to the constraint (3.19), $\dot{a} = 0$ is inconsistent with positive energy density. Non-singular solutions could then take the form of a forever expanding or contracting spacetime. Clearly this type of behavior is only realizable if the dominating matter component in the system violates the SEC in the asymptotic region where the volume of the spacetime becomes small. This can almost immediately be excluded however since, as shown in section 4.1, as $a \rightarrow 0$ the components violating the SEC become completely negligible with respect to the other sources. These non-bouncing solutions would perhaps be more relevant for the gravitational physics of unstable branes henceforth referred to as half-s-branes.

The supergravity action associated with the s8-brane is the expression (5.2) with $m = 9 = p$ and $a = -5/2$. The corresponding equations of motion are equivalent to that of a single scalar field (the dilaton since there are not transverse dimensions) coupled to gravity with a positive potential of the form (4.1) where $\alpha = -5/2$. Λ is then a parameter corresponding to the magnitude of the 9-form flux. In sections 4 and 5 we have considered explicitly only solutions with positive α . The solutions are unaltered for negative α but the distinction is important in string theory. In fact because α is negative for the s-branes of interest, the geometries would extrapolate between vacua ($\phi \rightarrow \pm\infty$) where the string coupling is very small ($g_s = e^{2\phi}$).

The s8-brane solutions were already studied in section 4. We found that the $k = 0$ solutions are always singular either in the past (big-bang solution) or in the future (big-crunch solution). For the spherical s8-brane, bounces actually occur but since $|a| = 5/2 > \alpha_c$ the corresponding solutions are singular both in the past and the future of the bounce. In other words it is the matter components respecting the SEC that eventually comes to dominate before and after the bouncing region. The apparition of singular points is then predicted by the singularity theorems because the latter sources always succeed in driving the spacetimes into a phase of contraction (expansion) sometime in the future (past).

Refs. [16, 17] suggested that the singularities found above could be resolved by introducing in the system the most relevant open string degree of freedom namely the tachyon. The latter couples to gravity and the dilaton with a term of the form (see [17] and references therein)

$$S_{brane} = -T_p \int d^{p+1}x e^{-\phi} V(T) \sqrt{-|\mathcal{P}G_{ab} + \partial_a T \partial_b T|} \delta(\mathbf{y}), \quad (6.3)$$

where $V(T) = 1/\cosh(T/\sqrt{2})$, \mathcal{P} is the pullback and the delta function localizes the unstable object in the transverse spatial directions labelled \mathbf{y} . The form of the potential and the regime of validity associated with the action (6.3) are considered in ref. [27]. This minimum extension consisting in coupling the tachyon to the massless closed string modes²⁹ was doomed from the start. In fact it is shown in ref. [37] that the source (6.3) respects the SEC for $p \leq 6$. Based on the analysis presented in this paper it is clear that such a contribution cannot change the singular outcome associated with the supergravity s-branes found in refs. [14, 15].

However for $p = 6$ and $p = 7$ the tachyon typically favors a short period of positive acceleration whenever its time-derivative is close to zero. Typically the tachyon extrapolates from a region where $\dot{T} \approx 0$ (when the tachyon is close or at the top of the potential $V(T)$) toward a late time asymptotic state (tachyon matter) where $\dot{T} \rightarrow 1$ [17]. The phase of positive acceleration can therefore occur only for a short time after the tachyon starts rolling down. This can certainly act as a gravitational source for a bounce. However for the spherical s8-brane this cannot be used to resolve the big-crunch and big-bang singularities we found earlier because the period of positive acceleration is too short. Even worst, the tachyon contribution at late and early time favors gravitational collapse. Similar pessimistic comments apply to the s7-brane. However a case deserving further study is that of the $k = 0$ s7- and s8-branes coupled to the tachyon. It is then conceivable that the $\dot{T} \approx 0$ region could be associated with the singular $t \rightarrow -\infty$ (for the big-bang solution) region. This may actually insure a safe landing (as $t \rightarrow -\infty$) for the scale factor since the cosmological singularity theorems would not apply there.

6.2 Unstable branes within branes

As our final result in this paper we describe a general mechanism by which the cosmological singularities associated with s-branes could be resolved. This applies both to the supergravity s-branes [13, 14, 15] and to s-brane gravity fields supported by a rolling tachyon [16, 17]. Before providing examples let us be more precise with respect to the nature of our claims by stating a conjecture:

Non-singular time-dependent spacetime solutions associated with supergravity s-branes can only exist if the evolution of the fields takes place in the presence of either stable D9-branes (D8-branes) in Type IIB (Type IIA) supergravity, or, a more elaborate configuration of lower-dimensional (perhaps smeared along some directions) branes such that the corresponding gravitational contribution violates the strong energy condition.

In other words we propose that, if the s-brane supergravity backgrounds are somehow dual to a tachyonic open string theory [45], only in cases for which the end point of the decay (and the point where brane creation begins) has non-zero vacuum energy can the gravitational backgrounds be non-singular. In fact we have seen in section 2.2 that tensile objects with spatial co-dimension zero or one are gravitational sources violating the SEC. Adding an ingredient like that to the usual supergravity system associated to s-branes [14, 15] at least invalidates our intuition based on the singularity theorems presented in section 2.3. In the remaining of this section we provide evidence that the conjecture actually holds.

²⁹This approach neglects the effect of inhomogeneities which are very much relevant (see ref. [44]).

6.2.1 Asymptotically dS non-singular s-branes

We begin by considering two very simple examples: (1) the Type IIB supergravity s7-brane in the presence of N stable D9-branes and, (2) the type IIA s8-brane in the presence of D8-branes. Evidence for the non-singular nature of the perhaps more interesting lower-dimensional branes is provided in the next section.

The action associated with the massless fields sourced by an unstable 7-brane is expression (5.2) with $m = 9$, $p = 8$ and $a = -2$. The contribution from the D9-branes is in the form of a ten-dimensional cosmological constant,

$$S_{D9} = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G} \Lambda_b, \quad (6.4)$$

with

$$\Lambda_b = \frac{N g_s}{(2\pi l_s)^3}. \quad (6.5)$$

The non-vanishing energy density is simply N times the D9-brane tension [43]. This new ingredient is precisely what is needed in order to resolve the IR (large $|t|$) singularities usually associated with s-branes.

We assume that the s7-brane has a metric ansatz of the form (6.1). The Friedmann constraint (3.19) becomes

$$\rho = 28 \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right], \quad (6.6)$$

from which the scale factor $R(t)$ has dropped out. For simplicity we consider that the spatial direction transverse to the unstable 8-brane is compactified on a circle of fixed size.³⁰ The effective $(8+1)$ -dimensional gravitational action is (5.5) with $p = 8$, $\psi = 0$ and $a = -2$, supplemented with the dimensionally reduced cosmological constant term (6.4),

$$S_\Lambda = -\frac{1}{16\pi G_N} \int d^9x \sqrt{-g} \Lambda_b. \quad (6.7)$$

The resulting system is simply gravity coupled to a scalar field with a positive potential of the form

$$V'(\phi) = \Lambda_b + \frac{C^2}{2} e^{2\phi}. \quad (6.8)$$

This corresponds to the potential illustrated on figure 5 with the sign of α changed and the $\phi \rightarrow -\infty$ region lifted to $+\Lambda_b$.

The gravitational backreaction of a scalar field rolling in a potential of the form (6.8) should lead to asymptotically dS spacetimes. The $k = +1$ solutions correspond to a 9-dimensional spacetime with a bounce separating asymptotic dS regions in the past (including the conformal boundary I^-) and in the future (with the boundary I^+). Figure 9 shows both the Penrose diagrams associated with a spherical s7-brane and with pure dS space in global coordinates. We consider the $k = 0$ at the end of this section.

The Type IIA s8-brane in the presence of D8-branes will lead to spacetimes which are qualitatively similar to those associated with the s7-branes. In Type IIA we consider an unstable 9-brane in the presence of N D8-branes smeared along the spatial transverse direction.

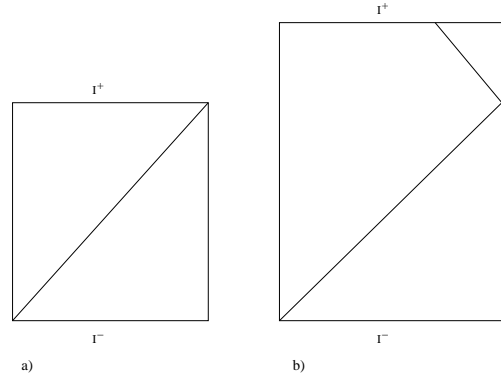


Figure 9: Conformal diagrams associated with a) pure dS space in global coordinates and b) the s7- and the s8-brane backgrounds.

³⁰The more general case with time-dependent $R(t)$ will be treated in ref. [41].

We now provide further support to the implicit assumption we made that spacetimes supported by scalar fields rolling in potentials of the form (6.8) lead to non-singular asymptotically dS spacetimes. Concrete applications will be considered in ref. [41]. In particular we study the $t > 0$ region following a bounce, the analysis being unchanged for $t < 0$. For the s7-brane ($p = 8$) the acceleration of the scale factor is provided by the expression

$$8\frac{\ddot{a}}{a} = -\frac{1}{2}\dot{\phi}^2 + \frac{1}{7}\left(\frac{C^2}{2}e^{2\phi} + \Lambda_b\right). \quad (6.9)$$

As pointed out in section 4.3 the time-reversal symmetric initial conditions at the bounce are $\dot{a}(0) = 0 = \dot{\phi}(0)$ and $\phi(0) = \phi_0$. The size of the unstable brane when it bounces is determined by solving the Friedmann constraint. There will be a phase of positive acceleration for $t \geq 0$. The question is whether or not it will last and, if not, whether the spacetime will be driven to gravitational collapse.

Curves II and IV on figure 3 represent typical effective equations of state associated with non-singular $k = +1$ solutions. A possibility is that the spacetimes of interest will have positive acceleration at all times (curve II). It is also conceivable that the initial phase of positive acceleration is followed by a period of negative acceleration where the kinetic energy of the scalar field (the only source not violating the SEC) comes to dominate. In section 4.3 we found many examples ($\Lambda_b = 0$) for which the spacetime never exits this phase (see curve III). Then the dominating scalar field contribution drives the spacetime into a phase contraction. Since from that point on the sources violating the SEC become increasingly negligible the singularity theorems apply and predict that singular points will appear in the future. A point we will be making is that for $\Lambda_b \neq 0$ contraction is unlikely to occur.

The equation of state for the scalar field associated with the s7-brane is

$$w = \frac{\dot{\phi}^2 - \frac{C^2}{2}e^{2\phi} - \Lambda_b}{\dot{\phi}^2 + \frac{C^2}{2}e^{2\phi} + \Lambda_b}. \quad (6.10)$$

For $\Lambda_b = 0$ the crux of the matter is which component ($\dot{\phi}^2$ or $(C^2/2)e^{2\phi}$) dominates for large $|\phi|$. If the kinetic energy term dominates then the asymptotic equation of state is close to $w = 1$ and, since this violates the SEC, singular points appear in the future. Non-singular solutions are those for which the potential term dominates in such a way that the SEC is violated in the future. This is clearly a very delicate system which appears to always be on the verge of developing singularities.

For $\Lambda_b \neq 0$ the situation is different. Inspecting eq. (6.10) we see that as long as $\dot{\phi}$ is kept small with respect to Λ_b the kinetic term will never dominate. It is possible that after the bounce ($t = 0$, $\dot{\phi} = 0$) the spacetime will enter a phase where the SEC is violated (negative acceleration) but it is likely to be temporary and lead to a state with positive acceleration in the asymptotic future. The gradient associated with the kinetic energy is

$$\frac{d}{dt}\dot{\phi} = -\left(p\dot{\phi}\frac{\dot{a}}{a} + C^2e^{2\phi}\right). \quad (6.11)$$

For $\Lambda_b = 0$ increasing C favors the potential term in the equation of state (6.10) but leads to an increase of $\ddot{\phi}$. This is why the $\dot{\phi}^2$ tends to dominate anyway in the future. However we see that for $\Lambda_b \neq 0$ the scalar field does not couple to the cosmological constant.³¹ Therefore by tuning the number of stable D-branes (N) it should be possible to find solutions where the vacuum energy contribution always dominates ($w = -1$) in the future.

³¹This is because the only potential-dependent expression appearing in the dilaton equation of motion is $\partial V/\partial\phi$.

For the $k = 0$ solutions a bounce is not allowed by the Friedmann constraint. The solutions are therefore either forever contracting or forever expanding. The corresponding non-singular solutions should be akin to the representation of dS space in terms of expanding inflationary coordinates (the corresponding conformal diagrams are described in ref. [55]). However we can show that no non-singular solutions exist in this case which is why they were not included in the conjecture. A very critical constraint associated with the systems of interest is that they respect the WEC ($w \geq -1$). For a scalar field (relevant for our s7- and s8-branes) this implies that

$$\frac{d\rho}{dt} = -2p\frac{\dot{a}}{a}\dot{\phi}^2. \quad (6.12)$$

This means the energy density,

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{C^2}{2}e^{2\phi} + \Lambda_b, \quad (6.13)$$

must decrease (increase) during a phase of expansion (contraction). However a non-singular $k = 0$ asymptotically dS solution must be associated with

$$\lim_{t \rightarrow \pm\infty} \left(\frac{1}{2}\dot{\phi}^2 + \frac{C^2}{2}e^{2\phi} \right) = 0. \quad (6.14)$$

This last condition is in direct contradiction with the WEC.

The WEC does not appear to put much constraint on the $k = 1$ solutions. However it will be important to verify that consistent scalar field dynamics exist for the s7- and s8-branes solutions described in this section [41].

6.2.2 The other s-branes

We now briefly explain why we expect the conjecture to hold for s-branes of all dimensions. We assume these gravitational objects are associated with a metric ansatz of the form (6.1) and that the volume of the 9-dimensional Cauchy surfaces goes through a bounce at $t = 0$. This region must be associated with a phase of positive acceleration. This is possible as long as the boundary conditions on the dilaton and the RR fields are chosen in such a way that the cosmological constant term dominates (on the RHS of the \ddot{a}/a expression) around the bounce (clearly this cannot happen when $\Lambda_b = 0$). Another condition at $t = 0$ for a bounce to occur is

$$p\frac{\dot{a}}{a} + (9-p)\frac{\dot{R}}{R} = 0. \quad (6.15)$$

Solutions with $\dot{a}(0) = 0$ and $\dot{R}(0) = 0$ can lead to a bounce but it is not the only option. This is fortunate because these boundary conditions exclude s-branes with the symmetries $k_1 = 0$ and $k_2 = -1$ proposed in ref [13] as is seen by inspection of the Friedmann constraint (3.19). In principle all combinations of k_1 and k_2 are allowed given appropriate boundary conditions for \dot{a}/a and \dot{R}/R .

A lesson we have learned in this work is that generating a gravitational bounce does not guarantee the whole spacetime is non-singular. In fact we have shown in section 4.3 that when there is no vacuum energy more often than not bouncing spacetimes are singular both in the past and the future. Our claim here is that the presence of a non-vanishing vacuum energy term (the stable branes) will resolve these big-bang and big-crunch singularities. The dilaton and the RR fields satisfy the SEC. We have shown in section 4.1 that for large values of the scale factor (in this case the average scale factor for the full ten-dimensional spacetime) a cosmological constant term always comes to dominate over contributions that respect the energy condition.

The dynamics of the hypothetical non-singular s-branes can be such that the acceleration of the spacetime is always positive, *i.e.*, the SEC is violated for all times. Another possibility is that

an s-brane solution evolving from $t = 0$ will traverse three phases. First it will get out of its initial phase of positive acceleration responsible for the bounce and enter a phase where the dilaton and RR fields dominate. It is possible that the spacetime will enter into a phase of contraction before the cosmological constant term becomes dominant again. The singularity theorems then predict singular points will develop in the future. However there should be a range of initial conditions allowing the s-branes to successfully enter a third phase which is vacuum-dominated in the future.

The non-singular s-brane solutions discussed here would be asymptotically dS₁₀. The issues related to the non-singular nature of the lower-dimensional s-branes will be fully addressed in ref. [41]. We believe it is likely the conjecture presented here will be proven. However a more conservative statement at this stage is that the no-go theorems presented in ref. [37, 38] are completely evaded by s-branes when they are cast in the more general and, perhaps, more realistic context presented here.

6.3 Open-closed duality and quantum effects

Let us close this section by making a few comments regarding the relevance of supergravity s-branes for the physics of unstable D-branes. According to the open-closed string duality conjectured by Sen [45] we should consider unstable D-branes (at least at tree-level) from the point of view of either closed strings or open strings, not both. Perhaps this can be interpreted to mean that studying s-branes in a supergravity context with the massless closed string modes sourced by the open string tachyon is not the correct thing to do [17]. Another problem for the supergravity approach is that as the open string tachyon rolls it emits massive closed strings, the contribution of which is not negligible [46]. For example unstable D0-branes decay essentially entirely into massive closed strings. This result is based on the tree-level calculation of the amplitude for the emission of a closed string mode in a theory with appropriate marginal tachyon boundary operator inserted. The integration over kinetically allowed modes (*i.e.*, those with $m^2 \leq g_s^{-1}$) leads to a divergent result. This is interpreted as tachyon matter [26] evaporation into massive closed strings. For unstable p -branes with $p > 1$ the presence of inhomogeneous tachyon degrees of freedom is likely to affect significantly the physics and results obtained assuming homogeneity must be used with caution. It is nevertheless interesting to note that for $p > 2$ the integration over massive modes leads to finite results [46]. However for branes with a characteristic size of the order $a_0 \sim l_s$ the total amplitude is again divergent while for large enough branes the result is finite. We use this as a hint that massive closed string modes emission can be made negligible by considering unstable branes of appropriate size. This could, in principle, justify using the supergravity approximation, *i.e.*, considering only the massless closed string modes when studying these objects.

7. Discussion

Refs. [29, 30, 31] pointed out that positive acceleration effects in $(3+1)$ -dimensions can be obtained from ten- and eleven-dimensional supergravity. This led to a renewed interest in cosmological solutions associated with the low energy dynamics of string theories [32, 33, 47, 48, 49]. Our analysis is complementary and focuses on studying (and trying to resolve) the pervading singularities associated with super-gravitational time-dependent backgrounds. We examined the possibility of obtaining bounces in theories associated with very simple dimensional reductions of supergravity theories. Considering flux compactifications on maximally symmetric Euclidean spaces we find a negative result for bouncing solutions in the effective $(p+1)$ -dimensional gravitational theory. For the non-singular bounces presented in section 4.3 we found they cannot be uplifted to ten or eleven dimensions. This does not exclude that an embedding could be found in the context of a richer compactification scheme, *i.e.*, by considering transverse manifolds associated with more

interesting symmetries (see, *e.g.*, ref. [32]). The only bouncing solutions we found that can be embedded in higher-dimensional theories are those containing both a big-crunch and a big-bang singularity. It is possible that one of these singularities is only an illusion of the compactification scheme used. In fact it is likely the dynamics of the breathing mode will in effect resolve one of those singularities. The other type of effective solutions considered is referred to as the singular *infinite throat*. In particular let us consider the big-crunch throat ($s = +1$ in section 4.2) possessing a curvature singularity in their future. For $p \leq 5$ ($d = 10$) and $p \leq 6$ ($d = 11$), the singularity structure remains qualitatively the same after the solution has been uplifted. However for other values of p the curvature singularity in the future remains but the higher-dimensional spacetime becomes singular in the past as well. This is an example of the phenomenon by which the regular nature of the lower-dimensional spacetime is only an illusion since singular points appear in the past as is seen by performing a higher-dimensional analysis. It is therefore clear that the concept of singularity resolution is ambiguous when considering effective cosmological solutions in the context of higher-dimensional theories.

So we provided an explicit example where a deceptively non-singular region in $(p + 1)$ dimensions is in fact associated with singular points if we consider its embedding in a higher-dimensional theory. More generally it should be possible to find effective theories (obtained from compactification of a higher-dimensional theory) admitting non-singular gravitational solutions containing a bounce. (The explicit big-crunch example we provided can be regarded as exemplifying the behavior in the past of such a solution.) Based on the analysis performed in section 5 we can extrapolate what the higher-dimensional behavior of such a solution might be. The cosmological singularity theorems predict that the higher-dimensional spacetime must contain singular points,³² and we assume these are in the past. For the future of these solutions there is really nothing special to say. Whatever lower-dimensional cosmological evolution is found should be what is detected by a $(p + 1)$ -dimensional observer.³³ The past will be more subtle and interesting. The $(p + 1)$ -dimensional observer would observe in her past a bounce, *i.e.*, a region where its universe becomes very small. She would probably conclude that this region is associated with singular points where quantum gravitational effects become important. However this is not the case assuming of course that the volume of the higher-dimensional spacetime is large enough. This phenomenon is an example where the dynamics of the transverse dimensions resolve a spacelike singularity. Of course this state of affair would only be temporary since the spacetime *must* contain singular points further in the past based on the cosmological singularity theorems. This situation might be interesting for cosmology because it can push back in the past (perhaps very far) the time where issues of quantum gravity become important. This, for example, would be relevant if the breathing mode plays the role of the inflaton. Of course up to now there was not very much success in deriving potentials from string theory leading to realistic models of inflation.

We have also considered a type of solutions corresponding to Lorentzian wormholes. These are the static equivalent of bouncing cosmologies. These geometries are associated with two disconnected Lorentzian boundaries. An important $(3 + 1)$ -dimensional result found in refs. [10, 11] is that such spacetimes can only exist in theories containing sources violating the WEC. This result is obtained by performing a local analysis. When global aspects are considered it is found that

³²It is important to recall that we are assuming the higher-dimensional theory (in this case supergravity) satisfies the conditions in these theorems.

³³There is of course the issue of making sure that physical predictions are not affected too much in the resulting theory if we consider $p = 3$ spacetimes. For example, the gravitational coupling must remain small enough to conform with experiments [50]. These issues were consistently ignored throughout our work. Also, the comments in this discussion are relevant in a braneworld'ish context where matter as we know it is confined on the $(p + 1)$ -dimensional manifold.

static spacetimes with disconnected boundaries exist (even if the WEC is satisfied) but that the boundaries must be separated by an horizon. This conclusion is reached by proving topological censorship theorems [12]. A Schwarzschild black hole is a spacetime with causally disconnected boundaries.

We generalized these results to higher-dimensional spacetimes and found the main conclusion to be unchanged. In fact reproducing the analysis found in section 3 for wormholes we find that the NEC must be violated for traversable wormhole throats (*i.e.*, a spacetime with causally related disconnected boundaries) to exist. However it is interesting to wonder whether or not it is possible to find traversable wormhole geometries in the context of $(p+1)$ -dimensional effective theories obtained by compactification of higher-dimensional theories. The main obstruction, as compared to the case of bouncing cosmologies, is that it is typically much harder to violate the NEC ($\rho+p \geq 0$) than to violate the SEC ($(n-2)\rho+np \geq 0$). For example a scalar field ϕ in a potential $V(\phi)$ is such that $\rho+p = \dot{\phi}^2/2$ which is clearly positive-definite. A possible way to obtain lower-dimensional scalar field models associated with traversable wormhole solutions is if the resulting effective theory contains a curvature coupling terms of the right sign. Even if we can find such effective theories it is guaranteed, based on the topological theorems, that an horizon will appear as seen from the higher-dimensional spacetime point of view.

If they exist it would be very interesting to cast Lorentzian wormholes in a gauge/gravity duality context. An example of non-traversable wormhole is the AdS_3 eternal black hole. In this case the conformal field theories on the two boundaries (separated by an horizon) were argued in ref. [51] to be in an entangled state. In an asymptotically AdS traversable wormhole there would be two causally connected maximally symmetric boundaries. The asymptotia would both have boundaries with conformal group isometries. This is reminiscent of the renormalization group flow interpretation of asymptotically anti-de Sitter spacetimes in AdS/CFT. However the situation would be different since in the RG flow picture one of the fixed points (IR) is not a boundary [52]. The presence of two causally connected boundaries should be closely related to cases where a time-dependent background is conjectured to be dual to Euclidean field theories on spacelike boundaries. An attempt to find such a duality is the dS/CFT correspondence [53]. In this case the gravitational background is either pure or asymptotically dS space [54, 55].

Acknowledgments

I would like to thank Christian Armendariz-Picon, Sean Carroll, Damien Easson, Aki Hashimoto, Rajesh Govindan, Stefan Hollands, Alex Maloney, Shiraz Minwalla, Vasilis Niarchos, Kazumi Okuyama, Amanda Peet, Daniel Robbins, Sav Sethi, Gary Shiu and Robert Wald for useful conversations. Part of this research was supported by NSERC of Canada.

A. Scalar field inter-breeding

In section 5 we considered $(p+1)$ -dimensional gravitational models obtained from compactification of higher-dimensional theories. Only solutions with the dilaton turned off were studied. Here we consider in some detail the case where both the breathing mode and the dilaton are excited. The effective potential is the of the form

$$V(\phi, \psi) = \frac{C^2}{2} e^{-\left(a\phi + \sqrt{\frac{2np^2}{(p-1)(n+p-1)}}\right)\psi} - \sigma n(n-1) e^{-\sqrt{\frac{2(n+p-1)}{n(p-1)}}\psi}. \quad (\text{A.1})$$

The solutions with $k=0$ (the spatial curvature on the $(p+1)$ -dimensional Lorentzian sub-manifold) and $\sigma = -1$ are the supergravity s-brane solutions found in refs. [13, 14, 15]. Our interest lies in

finding $(p+1)$ -dimensional solutions containing a bounce. We have since in section 5 that simple truncations to one scalar field do not lead to regular solutions. In this section we investigate whether or not the dilaton field can be used to resolve the singularities.

We consider the simple case $\sigma = 0$, *i.e.*, the curvature of the transverse dimensions vanishes. Using the metric ansatz (4.16) with the gauge $A = pB$ the equations of motion are

$$\ddot{B} = \frac{1}{p-1} e^{2pB-\alpha\psi-a\phi} - k(p-1)e^{2(p-1)B}, \quad (\text{A.2})$$

$$\ddot{\phi} = ae^{2pB-\alpha\psi-a\phi}, \quad (\text{A.3})$$

$$\ddot{\psi} = \alpha e^{2pB-\alpha\psi-a\phi}, \quad (\text{A.4})$$

where $\alpha = \sqrt{\frac{2np^2}{(p-1)(n+p-1)}}$ and $a = (4-p)/2$. The Friedmann constraint takes the form

$$\frac{p(p-1)}{2} \left(\dot{B}^2 + ke^{2(p-1)B} \right) = \frac{1}{4} \left(\dot{\phi}^2 + \dot{\psi}^2 \right) + \frac{C^2}{4} e^{2pB-\alpha\psi-a\phi}. \quad (\text{A.5})$$

By inspecting eqs. (A.3) and (A.4) we can write down the solution for the dilaton in terms of the breathing mode,

$$\phi(t) = \frac{a}{\alpha} \psi(t) + c_2 t + c_1, \quad (\text{A.6})$$

where c_1 and c_2 are constants of integration. Using this information the system of differential equations we need to solve takes the simpler form

$$\ddot{B} = \frac{\bar{C}^2}{2(p-1)} e^{2pB-\bar{\alpha}\psi-ac_1 t} - k(p-1)e^{2(p-1)B}, \quad (\text{A.7})$$

$$\ddot{\psi} = \frac{\alpha \bar{C}^2}{2} e^{2pB-\bar{\alpha}\psi-ac_1 t}, \quad (\text{A.8})$$

and the Friedmann constraint becomes

$$\frac{p(p-1)}{2} \left(\dot{B}^2 + ke^{2(p-1)B} \right) = \frac{1}{4} \left(\dot{\phi}^2 + \dot{\psi}^2 \right) + \frac{\bar{C}^2}{4} e^{2pB-\bar{\alpha}\psi-ac_1 t}, \quad (\text{A.9})$$

where we have defined

$$\bar{\alpha} = \frac{a^2 + \alpha^2}{\alpha}, \quad (\text{A.10})$$

and

$$\bar{C}^2 = C^2 e^{-ac_2}. \quad (\text{A.11})$$

Similarly to what we did in section 5 we consider bouncing spacetimes with the boundary conditions

$$\dot{\psi}(0) = 0 = \dot{B}(0), \quad \psi(0) = \psi_0. \quad (\text{A.12})$$

This implies that the boundary conditions for the dilaton are of the form

$$c_2 = \phi(0) - \frac{a}{\alpha} \psi_0, \quad c_1 = \dot{\phi}(0). \quad (\text{A.13})$$

There are two cases potentially leading to bouncing spacetimes. The simplest one consists in considering that the dilaton bounces simultaneously with the breathing mode and the gravitational field. This corresponds to the system with $c_1 = 0$. However we have already treated this case in section 4.3. We can therefore use the results found there by simply replacing α with $\bar{\alpha}$ and C with \bar{C} . We found earlier that all values of α obtained from string compactifications are such that $\alpha < \alpha_c$

and therefore lead to singular cosmologies. Since we always have $\bar{\alpha} > \alpha$ the same results apply to this simple dilaton-breathing mode system.

A potentially more interesting case consists in allowing for the kinetic energy stored in the dilaton field to be non-vanishing at the hypothetical bounce, *i.e.*, $c_1 \neq 0$. The Friedmann constraint at $t = 0$ then takes the form

$$c_1^2 = 2e^{2pB(0)} \left(p(p-1) - \frac{\bar{C}^2}{2} e^{-\alpha\bar{\psi}(0)} \right). \quad (\text{A.14})$$

We have performed numerous numerical experiments and our non-definite prediction is that the dilaton does not resolve the singularities associated with the bouncing spacetimes.

References

- [1] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], “Measurements of the Cosmological Parameters Ω and Λ from the First Seven Supernovae at $z \geq 0.35$,” *Astrophys. J.* **483**, 565 (1997) [arXiv:astro-ph/9608192];
S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], “Discovery of a Supernova Explosion at Half the Age of the Universe and its Cosmological Implications,” *Nature* **391**, 51 (1998) [arXiv:astro-ph/9712212];
A.G. Riess *et al.* [Supernova Search Team Collaboration], “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant,” *Astron. J.* **116**, 1009 (1998) [arXiv:astro-ph/9805201].;
N.A. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, “The Cosmic Triangle: Revealing the State of the Universe,” *Science* **284**, 1481 (1999) [arXiv:astro-ph/9906463].
- [2] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev. D* **68**, 046005 (2003) [arXiv:hep-th/0301240].
- [3] H. Liu, G. Moore and N. Seiberg, “Strings in a time-dependent orbifold,” *JHEP* **0206**, 045 (2002) [arXiv:hep-th/0204168]; H. Liu, G. Moore and N. Seiberg, “Strings in time-dependent orbifolds,” *JHEP* **0210**, 031 (2002) [arXiv:hep-th/0206182].
- [4] B. Craps, D. Kutasov and G. Rajesh, “String propagation in the presence of cosmological singularities,” *JHEP* **0206**, 053 (2002) [arXiv:hep-th/0205101]; M. Berkooz, B. Craps, D. Kutasov and G. Rajesh, “Comments on cosmological singularities in string theory,” *JHEP* **0303**, 031 (2003) [arXiv:hep-th/0212215].
- [5] S. R. Coleman, “Why There Is Nothing Rather Than Something: A Theory Of The Cosmological Constant,” *Nucl. Phys. B* **310**, 643 (1988).
- [6] I. R. Klebanov, L. Susskind and T. Banks, “Wormholes And The Cosmological Constant,” *Nucl. Phys. B* **317**, 665 (1989).
- [7] S. B. Giddings and A. Strominger, “String Wormholes,” *Phys. Lett. B* **230**, 46 (1989).
- [8] J. Maldacena and L. Maoz, “Wormholes in AdS,” arXiv:hep-th/0401024.
- [9] B. McInnes, “Quintessential Maldacena-Maoz cosmologies,” arXiv:hep-th/0403104.
- [10] M. S. Morris and K. S. Thorne, “Wormholes In Space-Time And Their Use For Interstellar Travel: A Tool For Teaching General Relativity,” *Am. J. Phys.* **56**, 395 (1988).

- [11] M. Visser, S. Kar and N. Dadhich, “Traversable wormholes with arbitrarily small energy condition violations,” *Phys. Rev. Lett.* **90**, 201102 (2003) [arXiv:gr-qc/0301003]; C. Barcelo and M. Visser, “Scalar fields, energy conditions, and traversable wormholes,” *Class. Quant. Grav.* **17**, 3843 (2000) [arXiv:gr-qc/0003025]; D. Hochberg and M. Visser, “The null energy condition in dynamic wormholes,” *Phys. Rev. Lett.* **81**, 746 (1998) [arXiv:gr-qc/9802048]; D. Hochberg and M. Visser, “Dynamic wormholes, anti-trapped surfaces, and energy conditions,” *Phys. Rev. D* **58**, 044021 (1998) [arXiv:gr-qc/9802046]; M. Visser and D. Hochberg, “Generic wormhole throats,” arXiv:gr-qc/9710001; M. Visser, “Traversable wormholes: The Roman ring,” *Phys. Rev. D* **55**, 5212 (1997) [arXiv:gr-qc/9702043]; M. Visser, “Traversable Wormholes: Some Simple Examples,” *Phys. Rev. D* **39**, 3182 (1989).
- [12] G. J. Galloway, K. Schleich, D. M. Witt and E. Woolgar, “Topological censorship and higher genus black holes,” *Phys. Rev. D* **60**, 104039 (1999) [arXiv:gr-qc/9902061]; “The AdS/CFT correspondence conjecture and topological censorship,” *Phys. Lett. B* **505**, 255 (2001) [arXiv:hep-th/9912119].
- [13] M. Gutperle and A. Strominger, “Spacelike branes,” *JHEP* **0204**, 018 (2002) [arXiv:hep-th/0202210].
- [14] C. M. Chen, D. V. Gal’tsov and M. Gutperle, “S-brane solutions in supergravity theories,” *Phys. Rev. D* **66**, 024043 (2002) [arXiv:hep-th/0204071].
- [15] M. Kruczenski, R. C. Myers and A. W. Peet, “Supergravity S-branes,” *JHEP* **0205**, 039 (2002) [arXiv:hep-th/0204144].
- [16] A. Buchel, P. Langfelder and J. Walcher, “Does the tachyon matter?,” *Annals Phys.* **302**, 78 (2002) [arXiv:hep-th/0207235].
- [17] F. Leblond and A. W. Peet, “SD-brane gravity fields and rolling tachyons,” *JHEP* **0304**, 048 (2003) [arXiv:hep-th/0303035].
- [18] G. Jones, A. Maloney and A. Strominger, “Non-Singular Solutions for S-branes,” arXiv:hep-th/0403050.
- [19] G. T. Horowitz and A. Strominger, “Black Strings And P-Branes,” *Nucl. Phys. B* **360**, 197 (1991); C. M. Hull and P. K. Townsend, “Unity of superstring dualities,” *Nucl. Phys. B* **438**, 109 (1995) [arXiv:hep-th/9410167]; P. K. Townsend, “The eleven-dimensional supermembrane revisited,” *Phys. Lett. B* **350**, 184 (1995) [arXiv:hep-th/9501068]; E. Witten, *Nucl. Phys. B* **443**, 85 (1995) [arXiv:hep-th/9503124];
- [20] A. Strominger, “Massless black holes and conifolds in string theory,” *Nucl. Phys. B* **451**, 96 (1995) [arXiv:hep-th/9504090].
- [21] C. W. Misner, K. S. Thorne and J. A. Wheeler, “Gravitation”, W. H. Freeman and Company, 1973.
- [22] R. M. Wald, “General Relativity,” The University of Chicago Press, 1984.
- [23] S. W. Hawking and G. F. R. Ellis, “The large scale structure of spacetime”, Cambridge University Press, 1973.
- [24] M. Spradlin, A. Strominger and A. Volovich, “Les Houches lectures on de Sitter space,” arXiv:hep-th/0110007.
- [25] A. Sen, “Non-BPS states and branes in string theory,” arXiv:hep-th/9904207.

- [26] A. Sen, “Rolling tachyon,” JHEP **0204**, 048 (2002) [arXiv:hep-th/0203211]. A. Sen, “Tachyon matter,” JHEP **0207**, 065 (2002) [arXiv:hep-th/0203265].
- [27] D. Kutasov and V. Niarchos, “Tachyon effective actions in open string theory,” Nucl. Phys. B **666**, 56 (2003) [arXiv:hep-th/0304045].
- [28] C. P. Burgess, F. Quevedo, R. Rabadan, G. Tasinato and I. Zavala, “Bouncing branes,” arXiv:hep-th/0310122.
- [29] P. K. Townsend, “Cosmic acceleration and M-theory,” arXiv:hep-th/0308149.
- [30] P. K. Townsend and M. N. R. Wohlfarth, “Accelerating cosmologies from compactification,” Phys. Rev. Lett. **91**, 061302 (2003) [arXiv:hep-th/0303097].
- [31] R. Emparan and J. Garriga, “A note on accelerating cosmologies from compactifications and S-branes,” JHEP **0305**, 028 (2003) [arXiv:hep-th/0304124].
- [32] E. Bergshoeff, A. Collinucci, U. Gran, M. Nielsen and D. Roest, “Transient quintessence from group manifold reductions or how all roads lead to Rome,” arXiv:hep-th/0312102.
- [33] L. Jarv, T. Mohaupt and F. Saueressig, “Quintessence Cosmologies with a Double Exponential Potential,” arXiv:hep-th/0403063.
- [34] A. Sen, “Remarks on tachyon driven cosmology,” arXiv:hep-th/0312153.
- [35] R. Brandenberger, D. A. Easson and D. Kimberly, “Loitering phase in brane gas cosmology,” Nucl. Phys. B **623**, 421 (2002) [arXiv:hep-th/0109165].
- [36] S. M. Carroll, J. Geddes, M. B. Hoffman and R. M. Wald, “Classical stabilization of homogeneous extra dimensions,” Phys. Rev. D **66**, 024036 (2002) [arXiv:hep-th/0110149].
- [37] A. Buchel and J. Walcher, “Comments on supergravity description of S-branes,” JHEP **0305**, 069 (2003) [arXiv:hep-th/0305055].
- [38] F. Leblond and A. W. Peet, “A note on the singularity theorem for supergravity SD-branes,” arXiv:hep-th/0305059.
- [39] J. E. Wang, “Twisting S-branes,” arXiv:hep-th/0403094.
- [40] G. Tasinato, I. Zavala, C.P. Burgess, F. Quevedo, “Regular S-Brane Backgrounds,” arXiv:hep-th/0403156.
- [41] F. Leblond and D. Robbins, work in progress.
- [42] K. Okuyama, “Wess-Zumino term in tachyon effective action,” JHEP **0305**, 005 (2003) [arXiv:hep-th/0304108].
- [43] J. Polchinski, “String Theory. Vol. 2: Superstring Theory And Beyond,” Cambridge University Press, 1998.
- [44] G. N. Felder, L. Kofman and A. Starobinsky, “Caustics in tachyon matter and other Born-Infeld scalars,” JHEP **0209**, 026 (2002) [arXiv:hep-th/0208019]; G. N. Felder and L. Kofman, “Inhomogeneous fragmentation of the rolling tachyon,” arXiv:hep-th/0403073.
- [45] A. Sen, “Open-closed duality at tree level,” Phys. Rev. Lett. **91**, 181601 (2003) [arXiv:hep-th/0306137]; “Open-closed duality: Lessons from matrix model,” arXiv:hep-th/0308068.
- [46] N. Lambert, H. Liu and J. Maldacena, “Closed strings from decaying D-branes,” arXiv:hep-th/0303139.

- [47] L. Jarv, T. Mohaupt and F. Saueressig, “Quintessence cosmologies with a double exponential potential,” arXiv:hep-th/0403063.
- [48] I. P. Neupane, “Accelerating cosmologies from exponential potentials,” arXiv:hep-th/0311071; I. P. Neupane, “Cosmic acceleration and M theory cosmology,” arXiv:hep-th/0402021.
- [49] M. N. R. Wohlfarth, “Accelerating cosmologies and a phase transition in M-theory,” Phys. Lett. B **563**, 1 (2003) [arXiv:hep-th/0304089]; N. Ohta, “A study of accelerating cosmologies from superstring/ M theories,” Prog. Theor. Phys. **110**, 269 (2003) [arXiv:hep-th/0304172]; C. M. Chen, P. M. Ho, I. P. Neupane and J. E. Wang, “A note on acceleration from product space compactification,” JHEP **0307**, 017 (2003) [arXiv:hep-th/0304177]; B. McInnes, “The strong energy condition and the S-brane singularity problem,” JHEP **0306**, 043 (2003) [arXiv:hep-th/0305107]; M. Ito, “On the solutions to accelerating cosmologies,” JCAP **0309**, 003 (2003) [arXiv:hep-th/0305130]; C. M. Chen, P. M. Ho, I. P. Neupane, N. Ohta and J. E. Wang, “Hyperbolic space cosmologies,” JHEP **0310**, 058 (2003) [arXiv:hep-th/0306291]; I. P. Neupane, “Inflation from string/M-theory compactification?,” Nucl. Phys. Proc. Suppl. **129**, 800 (2004) [arXiv:hep-th/0309139]; M. N. R. Wohlfarth, “Inflationary cosmologies from compactification,” Phys. Rev. D **69**, 066002 (2004) [arXiv:hep-th/0307179]; V. D. Ivashchuk, V. N. Melnikov and A. B. Selivanov, “Cosmological solutions in multidimensional model with multiple exponential potential,” JHEP **0309**, 059 (2003) [arXiv:hep-th/0308113].
- [50] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].
- [51] J. M. Maldacena, “Eternal black holes in Anti-de-Sitter,” JHEP **0304**, 021 (2003) [arXiv:hep-th/0106112].
- [52] H. Firouzjahi and F. Leblond, “The clash between de Sitter and anti-de Sitter space,” JCAP **0306**, 003 (2003) [arXiv:hep-th/0209248].
- [53] A. Strominger, “The dS/CFT correspondence,” JHEP **0110**, 034 (2001) [arXiv:hep-th/0106113].
- [54] A. Strominger, “Inflation and the dS/CFT correspondence,” JHEP **0111**, 049 (2001) [arXiv:hep-th/0110087].
- [55] F. Leblond, D. Marolf and R. C. Myers, “Tall tales from de Sitter space. I: Renormalization group flows,” JHEP **0206**, 052 (2002) [arXiv:hep-th/0202094].